

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2025 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with "*", which can be attempted as a test. For this test the time allocated in Mathematics, Physics and Chemistry are 25 minutes, 20 minutes and 25 minutes respectively.

FIITJEE

SOLUTIONS TO JEE (ADVANCED) – 2025 (PAPER-2)

Mathematics

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1 Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α , define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers x .

Then which one of the following statements is TRUE ?

(A) For $\alpha = 2$, $\lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(B) For $\alpha = 2$, $\lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

(C) For $\alpha = 3$, $\lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(D) For $\alpha = 3$, $\lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

Ans. C

Sol. $\alpha = 3$

$$\lim_{x \rightarrow x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \lim_{x \rightarrow x_0} \left| \frac{\frac{3xe^x + 3x - 3e^x - 3x}{3(e^x + 1)} + e^{x_0}}{x - x_0} \right|$$

$$\lim_{x \rightarrow x_0} \left| \frac{3xe^x + 3x - 3e^x - 3x - 3x_0e^x + 3e^{x_0}}{3(e^x + 1)(x - x_0)} \right|$$

$$\lim_{x \rightarrow x_0} \left| \frac{3e^x(x - x_0) - 3e^{x_0}(e^{x-x_0} - 1)}{3(e^x + 1)(x - x_0)} \right|$$

$$\lim_{x \rightarrow x_0} \left| \frac{e^x - e^{x_0}}{(e^x + 1)} \right| = 0$$

Q.2 Let R denote the set of all real numbers. Then the area of the region

$$\left\{ (x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0 \right\} \text{ is}$$

(A) $\frac{17}{16} - \log_e 4$

(B) $\frac{33}{8} - \log_e 4$

(C) $\frac{57}{8} - \log_e 4$

(D) $\frac{17}{2} - \log_e 4$

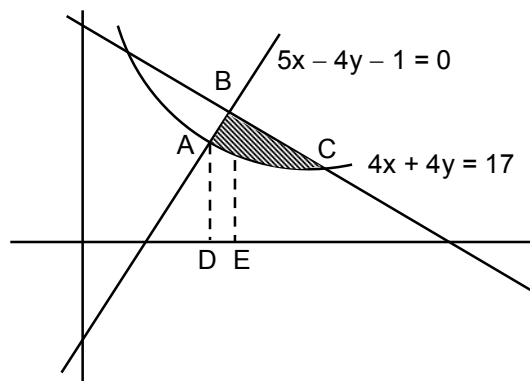
Ans. B

Sol. $x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0$

$B\left(2, \frac{9}{4}\right), A(1, 1), C\left(4, \frac{1}{4}\right)$

Required area

$$= \frac{1}{2} \left(1 + \frac{9}{4}\right) \times 1 - \int_1^4 \frac{1}{x} dx + \frac{1}{2} \left(\frac{9}{4} + \frac{1}{4}\right) \times 2 = \frac{33}{8} - \ln 4$$



*Q.3 The total number of real solutions of the equation

$$\theta = \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1} \left(\frac{6 \tan \theta}{9 + \tan^2 \theta} \right) \text{ is}$$

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively.)

(A) 1
(C) 3

(B) 2
(D) 5

Ans. C

Sol. $\theta = \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1} \left(\frac{6 \tan \theta}{9 + \tan^2 \theta} \right)$

$$= \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1} \left(\frac{2 \left(\frac{\tan \theta}{3} \right)}{1 + \left(\frac{\tan \theta}{3} \right)^2} \right)$$

Let $\frac{\tan \theta}{3} = \tan \alpha$

$$= \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1} \left(\frac{2 \tan \alpha}{1 + \tan^2 \alpha} \right)$$

$$= \tan^{-1}(2 \tan \theta) - \frac{1}{2} \sin^{-1}(\sin 2\alpha)$$

$$= \tan^{-1}(2 \tan \theta) - \tan^{-1} \left(\frac{\tan \theta}{3} \right)$$

$$\tan \theta = \frac{2 \tan \theta - \frac{\tan \theta}{3}}{1 + \frac{2 \tan^2 \theta}{3}} = \frac{5 \tan \theta}{3 + 2 \tan^2 \theta}$$

so $\tan \theta = 0$ or $\tan^2 \theta = 1$

$\theta = 0, \frac{\pi}{4}$ and $-\frac{\pi}{4}$

In another case we did not get any solution.

*Q.4 Let S denote the locus of the point of intersection of the pair of lines

$$4x - 3y = 12\alpha,$$

$$4\alpha x + 3\alpha y = 12,$$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points $(p, 0)$ and $(0, q)$, $q > 0$, and parallel to the line $4x - \frac{3}{\sqrt{2}}y = 0$.

Then the value of pq is

(A) $-6\sqrt{2}$

(B) $-3\sqrt{2}$

(C) $-9\sqrt{2}$

(D) $-12\sqrt{2}$

Ans. A

Sol. $4x - 3y = 12\alpha$... (1)

$$4\alpha x + 3\alpha y = 12$$

$$4x + 3y = \frac{12}{\alpha}$$
 ... (2)

$$16x^2 - 9y^2 = 144 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$y = mx \pm \sqrt{9m^2 - 16}$$

Now $q > 0$

$$\Rightarrow y = mx + \sqrt{9m^2 - 16}, m = \frac{4\sqrt{2}}{3}$$

$$\text{Hence } pq = \left(-\frac{\sqrt{9m^2 - 16}}{m} \right) (\sqrt{9m^2 - 16}) = -\frac{9m^2 - 16}{m} = -6\sqrt{2}$$

SECTION 2 (Maximum Marks: 16)

- This section contains **THREE (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
 choosing **ONLY** (A), (B) and (D) will get +4 marks;
 choosing **ONLY** (A) and (B) will get +2 marks;
 choosing **ONLY** (A) and (D) will get +2marks;
 choosing **ONLY** (B) and (D) will get +2 marks;
 choosing **ONLY** (A) will get +1 mark;
 choosing **ONLY** (B) will get +1 mark;
 choosing **ONLY** (D) will get +1 mark;
 choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
 choosing any other option(s) will get -2 marks.

Q.5 Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y and z for

which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$. Then which of the following statements is (are) TRUE?

- (A) The determinant of $Q - 2I$ is zero (B) The determinant of $Q - 6I$ is 12
 (C) The determinant of $Q - 3I$ is 15 (D) $yz = 2$

Ans. A, B

Sol. $QR = RP$

$$QR - 2R = RP - 2R$$

$$(Q - 2I)R = R(P - 2I) = R \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|(Q - 2I)R| = |R| |P - 2I| = 0$$

$$\text{For this to be true for all } R \Rightarrow |Q - 2I| = 0$$

(A) is correct.

$$QR = RP$$

$$QR - 3R = RP - 3R$$

$$(Q - 3I)R = R(P - 3I) = R \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$|Q - 3I| = 0$$

$$|Q - 2I| = 0$$

$$\Rightarrow 2(x - 2) = yz$$

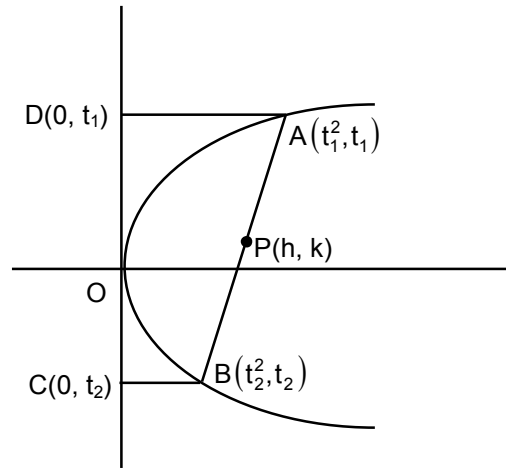
$$|Q - 3I| = 0 \Rightarrow x - 3 = yz$$

Hence $x = 1$, $yz = -2$

- *Q.6 Let S denote the locus of mid-points of those chords of the parabola $y^2 = x$, such that area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let R denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve S, and the lines $x = 1$ and $x = 4$. Then which of the following statements is (are) TRUE?
- (A) $(4, \sqrt{3}) \in S$ (B) $(5, \sqrt{2}) \in S$
 (C) Area of R is $\frac{14}{3} - 2\sqrt{3}$ (D) Area of R is $\frac{14}{3} - \sqrt{3}$

Ans. A, C

Sol. $2h = t_1^2 + t_2^2 \Rightarrow t_1 t_2 = 2k^2 - h$
 $2k = t_1 + t_2$
 Required area
 $= \frac{1}{2}(t_1^2 + t_2^2)(t_1 - t_2) - \frac{1}{3}(t_1^3 - t_2^3) = \frac{4}{3}$
 $\Rightarrow 3(t_1^2 + t_2^2)(t_1 - t_2) - 2(t_1^3 - t_2^3) = 8$
 $\Rightarrow (t_1 - t_2)[t_1 - t_2]^2 = 8$
 $\Rightarrow (t_1 - t_2)^2 = t_1^2 + t_2^2 - 2t_1 t_2$
 $= 2h - (4k^2 - 2h)$
 $4 = 4(h - k^2)$
 $y^2 = x - 1$
 Required area
 $= \int_1^4 (\sqrt{x} - \sqrt{x-1}) dx$
 $= \frac{2}{3} [x^{3/2} - (x-1)^{3/2}]_1^4 = \frac{14}{3} - 2\sqrt{3}$



- *Q.7 Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ such that $y_1 > 0$ and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point $(3, 0)$. Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where O denotes the origin $(0, 0)$. Let $|XY|$ denote the length of the line segment XY. Then which of the following statements is (are) TRUE?
- (A) The equation of the line joining P and Q is $2x + 3y = 3(1 + \sqrt{3})$
 (B) The equation of the line joining P and Q is $2x + y = 3(1 + \sqrt{3})$
 (C) If $N_2 = (x_2 = 0)$, then $3|N_2Q| = 2|N_2S|$
 (D) If $N_1 = (x_1 = 0)$, then $9|N_1P| = 4|N_1R|$

Ans. A, C

Sol. Coordinate of P : $\left(\frac{3\sqrt{3}}{2}, 1\right)$, R : $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

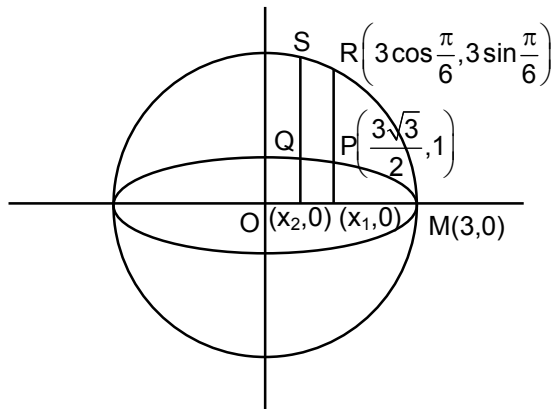
Q : $\left(\frac{3}{2}, \sqrt{3}\right)$, S : $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

Equation of line PQ : $y - \sqrt{3} = \frac{1 - \sqrt{3}}{\frac{3\sqrt{3}}{2} - \frac{3}{2}} \left(x - \frac{3}{2}\right)$

$\Rightarrow 2x + 3y = 3(1 + \sqrt{3})$

$\frac{N_2Q}{N_2S} = \frac{\sqrt{3}}{3 \frac{\sqrt{3}}{2}} = \frac{2}{3}$

Also $\frac{N_1P}{N_1R} = \frac{1}{3/2} = \frac{2}{3}$



Q.8 Let R denote the set of all real numbers. Let $f : R \rightarrow R$ be defined by

$$f(x) = \begin{cases} \frac{6x + \sin x}{2x + \sin x} & \text{if } x \neq 0 \\ \frac{7}{3} & \text{if } x = 0 \end{cases}$$

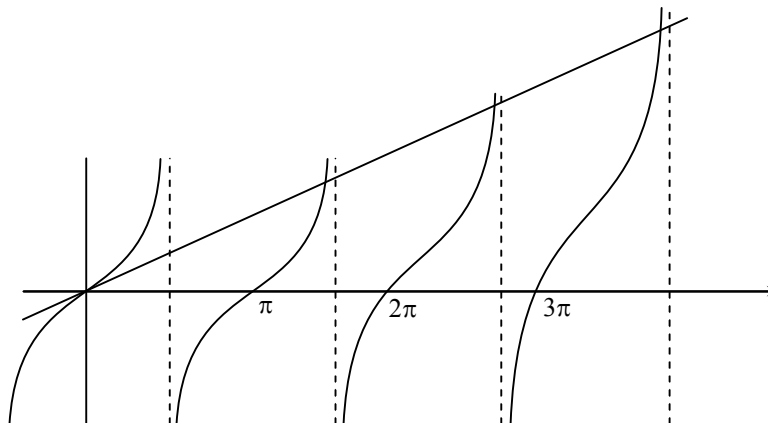
Then which of the following statements is (are) TRUE?

- (A) The point $x = 0$ is a point of local maxima of f
- (B) The point $x = 0$ is a point of local minima of f
- (C) Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3
- (D) Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

Ans. B, C, D

Sol. $f(x) = \frac{6x + \sin x}{2x + \sin x}, x \neq 0$
 $\frac{7}{3}, x = 0$

$f'(x) = \frac{4 \sin x - 4x \cos x}{(2x + \sin x)^2}, x \neq 0$
 $= \frac{4 \cos x (\tan x - x)}{(2x + \sin x)^2}, x \neq 0$



Now to check $x = 0$ is point of local maxima or minima

$$f'(0^-) < 0 \text{ and } f'(0^+) > 0$$

$\Rightarrow x = 0$ is point of local minima.

From the graph, we can say in the interval $[\pi, 6\pi]$ there is three point of local maxima and in the interval $[2\pi, 4\pi]$ there is one point of local minima.

SECTION 3 (Maximum Marks: 32)

- This section contains **SIX (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
Zero Marks : 0 In all other cases.

Q.9 Let $y(x)$ be the solution of the differential equation

$$x^2 \frac{dy}{dx} + xy = x^2 + y^2, x > \frac{1}{e},$$

satisfying $y(1) = 0$. Then the value of $2 \frac{(y(e))^2}{y(e^2)}$ is _____

Ans. 0.75

Sol. $x^2 \frac{dy}{dx} + xy = x^2 + y^2, x > \frac{1}{e}$ also $y(1) = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2 - xy}{x^2}$$

Put $y = vx$

$$\Rightarrow v + x \frac{dv}{dx} = v^2 - v + 1$$

$$\Rightarrow \frac{dv}{(v-1)^2} = \frac{dx}{x}$$

After integration

$$\Rightarrow \frac{-1}{v-1} = \ln x + c$$

$$\Rightarrow \frac{-x}{y-x} = \ln x + c$$

Now it is given $y(1) = 0$

$$\Rightarrow c = 1$$

$$\Rightarrow \frac{-x}{y-x} = \ln x + 1$$

$$\Rightarrow y = \frac{x \ln x}{1 + \ln x}$$

$$\Rightarrow \frac{2(y(e))^2}{y(e^2)} = \frac{3}{4} = 0.75$$

*Q.10 Let a_0, a_1, \dots, a_{23} be real numbers such that

$$\left(1 + \frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$

for every real number x . Let a_r be the largest among the numbers a_j for $0 \leq j \leq 23$. Then the value of r is _____

Ans. 6

Sol. a_0, a_1, \dots, a_{23} are real numbers and $\left(1 + \frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$

a_r be the largest among the numbers $a_j, 0 \leq j \leq 23$

\Rightarrow we have to find numerically greatest term.

$$m = \frac{n+1}{1 + \left|\frac{5}{2x}\right|} = \frac{48}{7} = 6.85$$

$([m] + 1)^{\text{th}}$ term is numerically greatest.

$\Rightarrow 7^{\text{th}}$ term is numerically greatest

$\Rightarrow r = 6$

Q.11 A factory has a total of three manufacturing units, $M_1, M_2,$ and $M_3,$ which produce bulbs independent of each other. The units $M_1, M_2,$ and M_3 produce bulbs in the proportions of $2 : 2 : 1,$ respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by $M_1,$ 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}.$

If a bulb is chosen randomly from the bulbs produced by $M_3,$ then the probability that it is defective is _____

Ans. 0.30

Sol. Let E_1 : Bulb is produced by M_1

E_2 : Bulb is produced by M_2

E_3 : Bulb is produced by M_3

A : produced bulb is defective

By using

$$P(A) = \sum_{i=1}^3 P(E_i)P\left(\frac{A}{E_i}\right)$$

$$= P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$\frac{20}{100} = \frac{2}{5} \cdot \frac{15}{100} + \frac{2}{5}P\left(\frac{A}{E_2}\right) + \frac{1}{5}P\left(\frac{A}{E_3}\right)$$

$$\Rightarrow \frac{7}{10} = 2P\left(\frac{A}{E_2}\right) + P\left(\frac{A}{E_3}\right) \quad \dots(1)$$

$$\begin{aligned} \text{Also, } P\left(\frac{E_2}{A}\right) &= \frac{2}{5} \\ \Rightarrow \frac{P(E_2 \cap A)}{P(A)} &= \frac{2}{5} \\ \Rightarrow \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(A)} &= \frac{2}{5} \\ \Rightarrow P\left(\frac{A}{E_2}\right) &= \frac{2 P(A)}{5 P(E_2)} = \frac{1}{5} \quad \dots(2) \end{aligned}$$

From eqn(1) and (2)

$$\begin{aligned} P\left(\frac{A}{E_3}\right) &= \frac{3}{10} \\ \Rightarrow \text{required probability} &= 0.30 \end{aligned}$$

Q.12 Consider the vectors

$$\vec{x} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{y} = 2\hat{i} + 3\hat{j} + \hat{k}, \quad \text{and} \quad \vec{z} = 3\hat{i} + \hat{j} + 2\hat{k}.$$

For two distinct positive real numbers α and β , define

$$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}, \quad \vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}, \quad \text{and} \quad \vec{Z} = \alpha\vec{z} + \beta\vec{x} = \vec{y}.$$

If the vectors \vec{X}, \vec{Y} , and \vec{Z} lie in a plane, then the value of $\alpha + \beta - 3$ is _____

Ans. -2

Sol. $[\vec{X} \vec{Y} \vec{Z}] = 0$

$$\begin{aligned} \Rightarrow [\vec{x} \vec{y} \vec{z}] \begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix} &= 0 \\ \Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} \begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix} &= 0 \\ -18(\alpha(\alpha^2 + \beta) - \beta(-\alpha - \beta^2) - (1 - \alpha\beta)) &= 0 \\ \Rightarrow \alpha^3 + \beta^3 - 1 + 3\alpha\beta &= 0 \\ \text{since } \alpha \neq \beta & \\ \Rightarrow \alpha + \beta + (-1) &= 0 \\ \text{So, } \alpha + \beta - 3 &= -2. \end{aligned}$$

*Q.13 For a non-zero complex number z , let $\arg(z)$ denote the principal argument of z , with $-\pi < \arg(z) \leq \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right).$$

Then the value of $\frac{3\alpha}{\pi}$ is _____

Ans. -2

Sol. $-\pi < \arg z \leq \pi$
 $0 < \arg w < \pi$

$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)$$

Now, $\sum_{n=1}^{2025} = -\omega \left(\frac{(-\omega)^{2025} - 1}{-\omega - 1}\right)$

$$= \frac{-2\omega}{1+\omega} = 2\omega^2$$

$$= 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1 - \sqrt{3}i$$

$$\alpha = (-1 - \sqrt{3}i) = \frac{-2\pi}{3}$$

$$\Rightarrow \frac{3\alpha}{\pi} = -2$$

Q.14 Let R denote the set of all real numbers. Let $f : R \rightarrow R$ and $g : R \rightarrow (0, 4)$ be functions defined by

$$f(x) = \log_e(x^2 + 2x + 4), \text{ and } g(x) = \frac{4}{1 + e^{-2x}}.$$

Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = f(g^{-1}(x))$, where g^{-1} is the inverse of the function g .

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is _____

Ans. 0.25

Sol. Assume $g^{-1}(x) = h(x)$

$$h : (0, 4) \rightarrow R$$

$$h(x) = -\frac{1}{2}[\log(4-x) - \log x]$$

$$f(h) = \log(h^2 + 2h + 4) \quad \forall 0 < h < 4$$

$$f'(h(x)) = \frac{1(2h(x) + 2)h'(x)}{h^2(x) + 2h(x) + 4}$$

$$f'(h(2)) = \frac{(2h(2) + 2)h'(2)}{(h^2(2) + 2h(2) + 4)}$$

$$\text{Here } h(2) = 0 \text{ and } h'(2) = \frac{1}{2}$$

$$\Rightarrow f'(g^{-1}(x)) \text{ at } x = 2 = \frac{\frac{1}{2}(2)}{4} = 0.25$$

*Q.15 Let

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \dots + \frac{1}{\sin 118^\circ \sin 119^\circ}$$

Then the value of

$$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2$$

is _____

Ans. 3

Sol.
$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \dots + \frac{1}{\sin 118^\circ \sin 119^\circ}$$

$$= (\tan 30^\circ - \tan 29^\circ)$$

$$+ (\tan 28^\circ - \tan 27^\circ)$$

$$\vdots$$

$$\vdots$$

$$+ (\tan 0^\circ - \tan 1^\circ)$$

$$\vdots$$

$$\vdots$$

$$+ (-\tan 26^\circ + \tan 27^\circ)$$

$$\Rightarrow (-\tan 28^\circ + \tan 29^\circ)$$

$$\alpha = \frac{\tan 30^\circ}{\sin 1^\circ} = \frac{1}{\sqrt{3}} \operatorname{cosec} 1^\circ$$

$$\Rightarrow \left(\frac{\operatorname{cosec} 1^\circ}{\alpha} \right)^2 = 3$$

Q.16 If

$$\alpha = \int_{1/2}^2 \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx,$$

then the value of $\sqrt{7} \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$ is _____

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Ans. 21

Sol.
$$\alpha = \int_{1/2}^2 \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx$$

Put $z = \frac{1}{x}$

$$\alpha = \int_2^{1/2} \frac{\cot^{-1} z (-dz)}{2z^2 - 3z + 2}$$

$$\alpha = \int_{1/2}^2 \frac{\left(\frac{\pi}{2}\right) - \tan^{-1}(z)}{2z^2 - 3z + 2}$$

$$2\alpha = \frac{\pi}{2} \int_{1/2}^2 \frac{1}{2z^2 - 3z + 2}$$

$$2\alpha = \frac{\pi}{2} \times \frac{1}{2} \int_{1/2}^2 \frac{1 \cdot dz}{\left(z - \frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$\Rightarrow \text{Now using } \int \frac{1}{x^2 + a^2} dx$$

$$= \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

We have

$$2\alpha = \frac{\pi}{\sqrt{7}} \left[\tan^{-1} \frac{5}{\sqrt{7}} + \tan^{-1} \frac{1}{\sqrt{7}} \right]$$

$$\Rightarrow \frac{2\alpha\sqrt{7}}{\pi} = \tan^{-1}(3 \times \sqrt{7})$$

$$\tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right) = 3 \times \sqrt{7}$$

$$\text{Now, } \sqrt{7} \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right) = 21$$

Physics

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
 Full Marks : +3 If **ONLY** the correct option is chosen;
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
 Negative Marks : -1 In all other cases.

*Q.1 A temperature difference can generate e.m.f. in some materials. Let S be the e.m.f. produced per unit temperature difference between the ends of a wire, σ the electrical conductivity and κ the thermal conductivity of the material of the wire. Taking M , L , T , I and K as dimensions of mass, length, time, current and temperature, respectively, the dimensional formula of the quantity

$$Z = \frac{s^2 \sigma}{\kappa} \text{ is:}$$

$$(A) [M^0 L^0 T^0 I^0 K^0]$$

$$(B) [M^0 L^0 T^0 I^0 K^{-1}]$$

$$(C) [M^1 L^2 T^{-2} I^{-1} K^{-1}]$$

$$(D) [M^1 L^2 T^{-4} I^{-1} K^{-1}]$$

Ans. B

Sol.

$$S = \frac{\text{emf}}{T}$$

$$R = \frac{1}{\sigma} \cdot \frac{L}{A}$$

$$\sigma = \frac{LI}{A \cdot \text{emf}}$$

$$\frac{J}{\text{sec}} = KA \frac{\Delta T}{L}$$

$$k = \frac{JL}{A \cdot \Delta T \text{sec}}$$

$$z = \frac{s^2 \sigma}{k}$$

$$= \frac{(\text{emf})^2}{T^2} \cdot \frac{LI}{A(\text{emf})} \times \frac{A(\Delta T)\text{sec}}{JL}$$

$$= \text{emf} \cdot \frac{l(\text{sec})}{T \cdot J}$$

$$= \frac{J}{l\text{sec}} \cdot \frac{l \cdot \text{sec}}{T \cdot J}$$

$$\text{Dimensional formula of } Z = [M^0 L^0 T^0 I^0 K^{-1}]$$

$T \rightarrow$ Temperature

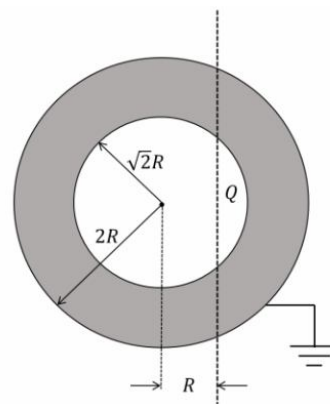
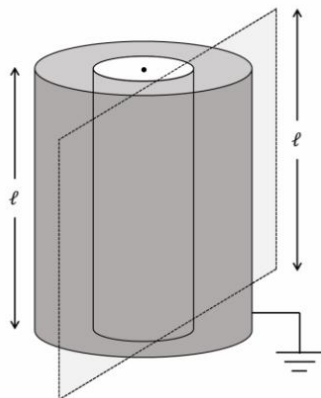
$A \rightarrow$ Area

$L \rightarrow$ Length

$J \rightarrow$ Joule

$l \rightarrow$ Current

Q.2 Two co-axial conducting cylinders of same length ℓ with radii $\sqrt{2}R$ and $2R$ are kept, as shown in Fig. 1. The charge on the inner cylinder is Q and the outer cylinder is grounded. The annular region between the cylinders is filled with a material of dielectric constant $\kappa = 5$. Consider an imaginary plane of the same length ℓ at a distance R from the common axis of the cylinders. This plane is parallel to the axis of the cylinders. The cross-sectional view of this arrangement is shown in Fig. 2. Ignoring edge effects, the flux of the electric field through the plane is (ϵ_0 is the permittivity of free space):



- (A) $\frac{Q}{30\epsilon_0}$
 (C) $\frac{Q}{60\epsilon_0}$

- (B) $\frac{Q}{15\epsilon_0}$
 (D) $\frac{Q}{120\epsilon_0}$

Ans. C

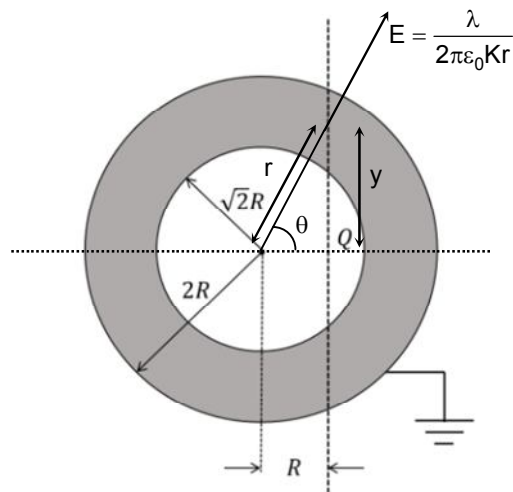
Sol.
$$d\phi_1 = \left(\frac{\lambda}{2\pi\epsilon_0\kappa} \right) (dy)\ell \cos\theta$$

$$= \frac{\lambda\ell}{10\pi\epsilon_0\kappa} (dy) \frac{R}{r} = \frac{\lambda\ell R}{10\pi\epsilon_0\kappa} \int_R^{\sqrt{3}R} \frac{dy}{(R^2 + y^2)}$$

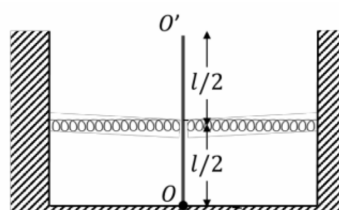
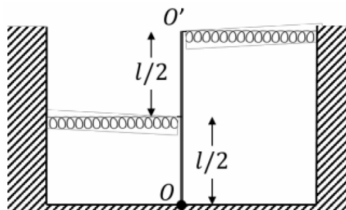
$$\phi_1 = \frac{\lambda\ell R}{10\pi\epsilon_0\kappa} \cdot \frac{1}{R} \tan^{-1}\left(\frac{y}{R}\right) \Big|_R^{\sqrt{3}R}$$

$$= \frac{\lambda\ell}{10\pi\epsilon_0\kappa} \times [\tan^{-1}(\sqrt{3}) - \tan^{-1}(1)]$$

$$= \frac{\lambda\ell}{10\pi\epsilon_0\kappa} \times \left(\frac{\pi}{3} - \frac{\pi}{4} \right) = \frac{\lambda\ell}{120\epsilon_0\kappa} = \frac{Q}{120\epsilon_0\kappa}$$
 Total flux = $2 \times \phi_1 = \frac{Q}{60\epsilon_0\kappa}$



*Q.3 As shown in the figures, a uniform rod OO' of length ℓ is hinged at the point O and held in place vertically between two walls using two massless springs of same spring constant. The springs are connected at the midpoint and at the top-end (O') of the rod, as shown in Fig.1 and the rod is made to oscillate by a small angular displacement. The frequency of oscillation of the rod is f_1 . On the other hand, if both the springs are connected at the midpoint of the rod, as shown in Fig. 2 and the rod is made to oscillate by a small angular displacement, then the frequency of oscillation is f_2 . Ignoring gravity and assuming motion only in the plane of the diagram, the value of $\frac{f_1}{f_2}$ is:



(A) 2

(C) $\sqrt{\frac{5}{2}}$

(B) $\sqrt{2}$

(D) $\sqrt{\frac{2}{5}}$

Ans. C

Sol. **Case I:** Restoring torque (When rod is rotated by θ)

$$\tau = k\ell\theta \cdot \ell + \left(k\frac{\ell}{2}\theta\right)\frac{\ell}{2} = \frac{5}{4}k\ell^2\theta$$

$$\tau = -\frac{m\ell^2}{3}\alpha$$

$$\Rightarrow \alpha = -\frac{15k}{4m}\theta$$

$$\Rightarrow f_1 = \frac{1}{2\pi}\sqrt{\frac{15k}{4m}} \quad \dots(i)$$

Case II: Restoring torque (When rod is rotated by θ)

$$\tau = \left(2k\frac{\ell}{2}\theta\right)\frac{\ell}{2} = \frac{k\ell^2\theta}{2}$$

$$\Rightarrow \frac{k\ell^2}{2}\theta = -I_0\alpha = -\frac{m\ell^2}{3}\alpha$$

$$\Rightarrow \alpha = -\frac{3k}{2m}\theta$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{3k}{2m}} \quad \dots(ii)$$

$$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{5}{2}}$$

*Q. 4 Consider a star of mass m_2 kg revolving in a circular orbit around another star of mass m_1 kg with $m_1 \gg m_2$. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg/s. In this transfer process, there is no other loss of mass. If the separation between the centers of the stars is r , then its relative rate of change $\frac{1}{r} \frac{dr}{dt}$ (in s^{-1}) is given by:

(A) $-\frac{3\gamma}{2m_2}$

(B) $-\frac{2\gamma}{m_2}$

(C) $-\frac{2\gamma}{m_1}$

(D) $-\frac{3\gamma}{2m_1}$

Ans. Close answer is B

Sol. Conserving angular momentum of
 $m_2 v_0 r = \text{constant}$

$$m_2 \sqrt{\frac{Gm_1}{r}} \cdot r = \text{constant}$$

Or $m_2^2 r = \text{constant}$

Differentiating $2m_2 \frac{dm_2}{dt} r + m_2^2 \frac{dr}{dt} = 0$

$$2m_2(-\gamma)r + m_2^2 \frac{dr}{dt} = 0 \Rightarrow \frac{1}{r} \frac{dr}{dt} = \frac{2\gamma}{m_2}$$

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:
 Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
 Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
 Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
 Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
 Negative Marks: -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
 choosing **ONLY** (A), (B) and (D) will get +4 marks;
 choosing **ONLY** (A) and (B) will get +2 marks;
 choosing **ONLY** (A) and (D) will get +2 marks;
 choosing **ONLY** (B) and (D) will get +2 marks;
 choosing **ONLY** (A) will get +1 mark;
 choosing **ONLY** (B) will get +1 mark;
 choosing **ONLY** (D) will get +1 mark;
 choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

- Q. 5 A positive point charge of 10^{-8} C is kept at a distance of 20 cm from the center of a neutral conducting sphere of radius 10 cm. The sphere is then grounded and the charge on the sphere is measured. The grounding is then removed and subsequently the point charge is moved by a distance of 10 cm further away from the center of the sphere along the radial direction. Taking $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ (where ϵ_0 is the permittivity of free space), which of the following statements is/are correct:
- (A) Before the grounding, the electrostatic potential of the sphere is 450 V .
 (B) Charge flowing from the sphere to the ground because of grounding is 5×10^{-9} C.
 (C) After the grounding is removed, the charge on the sphere is -5×10^{-9} C.
 (D) The final electrostatic potential of the sphere is 300 V .

Ans. A, B, C

Sol. Before grounding, potential at centre,

$$V_C = \frac{kq}{r} = \frac{9 \times 10^9 \times 10^{-8}}{0.2} = 45 \times 10 = 450 \text{ V}$$

After grounding, potential of sphere becomes zero. Let charge on sphere is Q

$$\frac{kQ}{0.1} + \frac{k \times 10^{-8}}{0.2} = 0$$

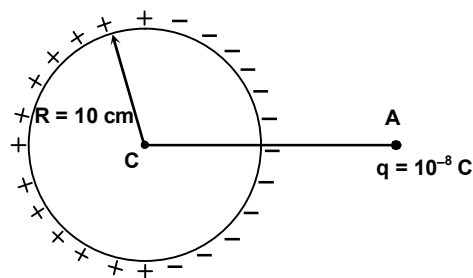
$$Q + \frac{10^{-8}}{2} = 0$$

\Rightarrow charge of sphere, $Q = -5 \times 10^{-9}$

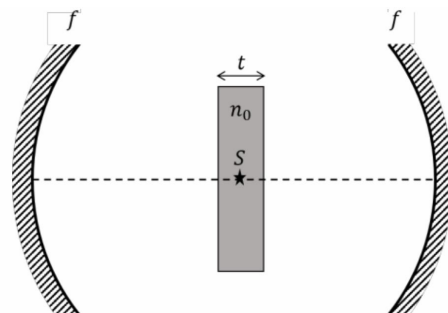
Final potential of sphere,

$$= \frac{9 \times 10^9 \times (-5 \times 10^{-9})}{0.1} + \frac{9 \times 10^9 \times 10^{-8}}{0.3}$$

$$= -450 + 300 = -150 \text{ V}$$



- Q.6 Two identical concave mirrors each of focal length f are facing each other as shown in the schematic diagram. The focal length f is much larger than the size of the mirrors. A glass slab of thickness t and refractive index n_0 is kept equidistant from the mirrors and perpendicular to their common principal axis. A monochromatic point light source S is embedded at the center of the slab on the principal axis, as shown in the schematic diagram. For the image to be formed on S itself, which of the following distances between the two mirrors is/are correct:



- (A) $4f + \left(1 - \frac{1}{n_0}\right)t$ (B) $2f + \left(1 - \frac{1}{n_0}\right)t$
 (C) $4f + (n_0 - 1)t$ (D) $2f + (n_0 - 1)t$

Ans. A, B

Sol. Shift in position of object due to refraction at surface of sheet = $\frac{t}{2} \left(1 - \frac{1}{n_0} \right)$

To make image at object itself O' must be at common focal point or common centre of curvature

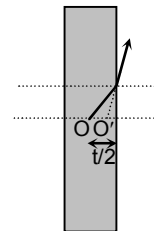
⇒ distance between mirrors

$$d_1 = f + f + 2 OO' = 2f + 2 \cdot \frac{t}{2} \left(1 - \frac{1}{n_0} \right)$$

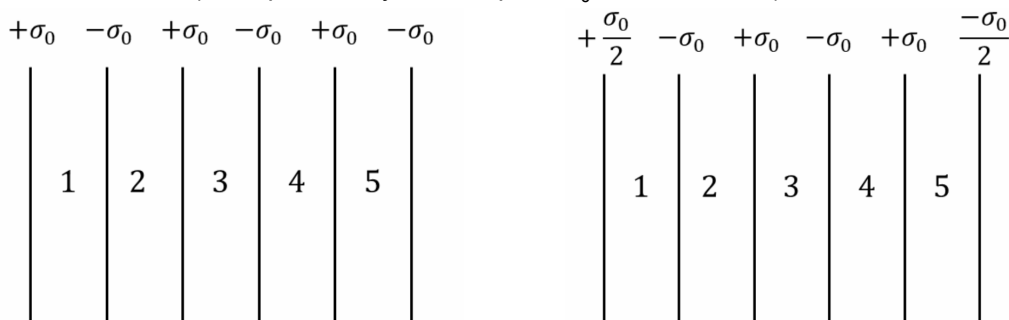
$$= 2f + t \left(1 - \frac{1}{n_0} \right)$$

$$d_2 = 2f + 2f + 2 OO'$$

$$= 4f + t \left(1 - \frac{1}{n_0} \right)$$



Q. 7 Six infinitely large and thin non-conducting sheets are fixed in configurations I and II. As shown in the figure, the sheets carry uniform surface charge densities which are indicated in terms of σ_0 . The separation between any two consecutive sheets is $1 \mu\text{m}$. The various regions between the sheets are denoted as 1, 2, 3, 4 and 5. If $\sigma_0 = 9 \mu\text{C}/\text{m}^2$, then which of the following statements is/are correct: (Take permittivity of free space $\epsilon_0 = 9 \times 10^{-12} \text{ F/m}$)



Configuration I

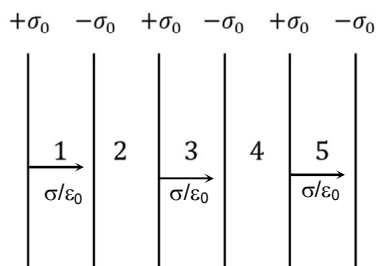
Configuration II

- (A) In region 4 of the configuration I, the magnitude of the electric field is zero.
- (B) In region 3 of the configuration II, the magnitude of the electric field is $\frac{\sigma_0}{\epsilon_0}$.
- (C) Potential difference between the first and the last sheets of the configuration I is 5 V.
- (D) Potential difference between the first and the last sheets of the configuration II is zero.

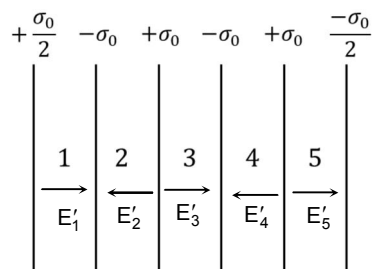
Ans. A

Sol. In region -4 of configuration-I

$$\vec{E}_4 = \frac{\sigma_0}{2\epsilon_0} - \frac{\sigma_0}{2\epsilon_0} + \frac{\sigma_0}{2\epsilon_0} - \frac{\sigma_0}{2\epsilon_0} - \frac{\sigma_0}{2\epsilon_0} + \frac{\sigma_0}{2\epsilon_0} = 0$$



Configuration I



Configuration II

In region -3 of configuration -II

$$E'_3 = \frac{\sigma_0/2}{2\epsilon_0} + \frac{\sigma_0/2}{2\epsilon_0} = \frac{\sigma_0}{2\epsilon_0}$$

In configuration -I

$$V_A = \frac{2\sigma_0}{2\epsilon_0} \times d - \frac{2\sigma_0}{2\epsilon_0} \times d - \frac{2\sigma_0}{2\epsilon_0} \times d = V_F$$

$$V_A - V_F = \frac{6\sigma_0}{2\epsilon_0} \times d = \frac{6 \times 9 \times 10^{-6}}{2 \times 9 \times 10^{-12}} \times 10^{-6} = 3 \text{ volt}$$

In configuration -II

$$E'_1 = \frac{\sigma_0}{4\epsilon_0} \times 2 = \frac{\sigma_0}{2\epsilon_0}$$

$$E'_2 = -\frac{2\sigma_0}{2\epsilon_0} + 2 \cdot \frac{\sigma_0/2}{2\epsilon_0} = -\frac{\sigma_0}{2\epsilon_0}$$

$$E'_3 = \frac{\sigma_0}{4\epsilon_0} \times 2 = \frac{\sigma_0}{2\epsilon_0}$$

$$E'_4 = -\frac{2\sigma_0}{2\epsilon_0} + \frac{2\sigma_0/2}{2\epsilon_0} = -\frac{\sigma_0}{2\epsilon_0}$$

$$E'_5 = \frac{\sigma_0}{2\epsilon_0}$$

$$V'_A - \frac{\sigma_0}{2\epsilon_0} d + \frac{\sigma_0}{2\epsilon_0} d - \frac{\sigma_0}{2\epsilon_0} d + \frac{\sigma_0}{2\epsilon_0} d - \frac{\sigma_0}{2\epsilon_0} d = V'_F$$

$$\Rightarrow V'_A - \frac{\sigma_0}{2\epsilon_0} d = V'_F$$

$$V'_A - V'_F = \frac{\sigma_0}{2\epsilon_0} d = \frac{9 \times 10^{-6}}{2 \times 9 \times 10^{-12}} \times 10^{-6} = \frac{1}{2} \text{ Volts}$$

*Q.8 The efficiency of a Carnot engine operating with a hot reservoir kept at a temperature of 1000 K is 0.4. It extracts 150 J of heat per cycle from the hot reservoir. The work extracted from this engine is being fully used to run a heat pump which has a coefficient of performance 10. The hot reservoir of the heat pump is at a temperature of 300 K. Which of the following statements is/are correct:

- (A) Work extracted from the Carnot engine in one cycle is 60 J .
 (B) Temperature of the cold reservoir of the Carnot engine is 600 K .
 (C) Temperature of the cold reservoir of the heat pump is 270 K .
 (D) Heat supplied to the hot reservoir of the heat pump in one cycle is 540 J .

Ans. A, B, C

Sol. Temperature of hot reservoir, $T_1 = 1000$ K

$$\therefore e = 1 - \frac{T_2}{T_1}$$

$$0.4 = 1 - \frac{T_2}{1000}$$

$$\Rightarrow \frac{T_2}{1000} = 0.6 \text{ or } T_2 = 600 \text{ K}$$

$$\text{Also } e = \frac{W}{Q_1} \Rightarrow 0.4 = \frac{W}{150}$$

Or $W = 60$ J

Coefficient of performance

$$\beta = \frac{Q'_2}{W} = \frac{1}{e} - 1$$

$$10 = \frac{1}{e} - 1$$

$$e = \frac{1}{11}$$

\Rightarrow Heat given to hot reservoir

$$Q'_1 = Q'_2 + W = 600 + 60 = 660 \text{ J}$$

$$e = 1 - \frac{T'_2}{T'_1}$$

$$\frac{1}{11} = 1 - \frac{T'_2}{300}$$

$$T'_2 = 272.22\text{K} \approx 270\text{K}$$

SECTION 3 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
 Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
 Zero Marks : 0 In all other cases.

- Q.9 A conducting solid sphere of radius R and mass M carries a charge Q . The sphere is rotating about an axis passing through its center with a uniform angular speed ω . The ratio of the magnitudes of the magnetic dipole moment to the angular momentum about the same axis is given as $\alpha \frac{Q}{2M}$. The value of α is _____

Ans. 1.67 (Range 1.65 to 1.70)

Sol. In case of shell

$$\frac{\text{magnetic moment}}{\text{Angular momentum}} = \frac{Q}{2M}$$

$$\frac{\mu}{L} = \frac{Q}{2M}$$

$$\text{Or } \mu = \frac{Q}{2M} \times \frac{2}{3} MR^2 \quad \dots(i)$$

$$\text{For solid conducting sphere, } \mu = \frac{Q}{2M} \cdot \frac{2}{3} MR^2$$

$$L = \frac{2}{5} MR^2 \omega$$

$$\frac{\mu}{L} = \frac{\frac{Q}{2M} \times \frac{2}{3} MR^2}{\frac{2}{5} MR^2} = \frac{5}{3} \left(\frac{Q}{2M} \right)$$

$$\Rightarrow \alpha = \frac{5}{3} = 1.67$$

- Q.10 A hydrogen atom, initially at rest in its ground state, absorbs a photon of frequency ν_1 and ejects the electron with a kinetic energy of 10 eV. The electron then combines with a positron at rest to form a positronium atom in its ground state and simultaneously emits a photon of frequency ν_2 . The center of mass of the resulting positronium atom moves with a kinetic energy of 5 eV. It is given that positron has the same mass as that of electron and the positronium atom can be considered as a Bohr atom, in which the electron and the positron orbit around their center of mass. Considering no other energy loss during the whole process, the difference between the two photon energies (in eV) is _____

Ans. 11.8

Sol. $h\nu_1 = (10 + 13.6) \text{ eV} = 23.6 \text{ eV} \quad \dots(i)$

$$\frac{1}{2} m v_e^2 = 10 \text{ eV} \quad \dots(ii)$$

\Rightarrow Total energy of positronium

$$E = \frac{1}{2} 2m \left(\frac{v_e}{2} \right)^2 + \left\{ \frac{1}{2} m \left(\frac{v_e}{2} \right)^2 + \frac{1}{2} 2m \left(\frac{v_e}{2} \right)^2 - \frac{9 \times 10^9 \times e \cdot e}{r} \right\}$$

(using Bohr's hypothesis)

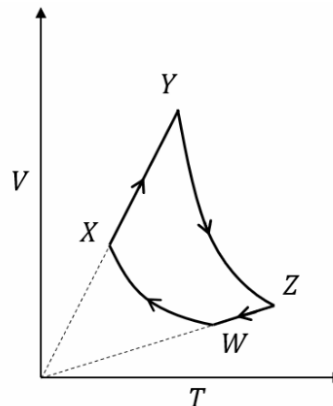
$$= (5 - 6.8) \text{ eV}$$

$$10 = 5 - 6.8 + h\nu_2$$

$$h\nu_2 = 11.8 \text{ eV}$$

$$h\nu_1 - h\nu_2 = 23.6 \text{ eV} - 11.8 \text{ eV} = 11.8 \text{ eV}$$

- *Q.11 An ideal monatomic gas of n moles is taken through a cycle $WXYZW$ consisting of consecutive adiabatic and isobaric quasi-static processes, as shown in the schematic V - T diagram. The volume of the gas at W , X and Y points are, 64 cm^3 , 125 cm^3 and 250 cm^3 respectively. If the absolute temperature of the gas T_W at the point W is such that $nRT_W = 1 \text{ J}$ (R is the universal gas constant), then the amount of heat absorbed (in J) by the gas along the path XY is _____



Ans. 1.6

Sol. Given $V_W = 64 \text{ cm}^3$
 $V_X = 125 \text{ cm}^3$
 $V_Y = 250 \text{ cm}^3$
 Process WX is adiabatic

$$T_W(64)^{\frac{5}{3}-1} = T_X(125)^{\frac{5}{3}-1}$$

$$\Rightarrow T_X = \frac{16}{25} T_W$$

Process XY is isobaric

$$\Rightarrow \frac{V_X}{V_Y} = \frac{T_X}{T_Y}$$

$$\frac{125}{250} = \frac{T_X}{T_Y}$$

$$T_Y = 2T_X$$

$$T_Y = \frac{32}{25} T_W$$

Heat absorbed in XY process

$$Q_{XY} = nC_P\Delta T = n \frac{5}{2} R(T_Y - T_X) = \frac{5}{2} nR \left(\frac{32}{25} T_W - \frac{16}{25} T_W \right) = \frac{8}{5} nRT_W = \frac{8}{5} \text{ J}$$

- *Q.12 A geostationary satellite above the equator is orbiting around the earth at a fixed distance r_1 from the center of the earth. A second satellite is orbiting in the equatorial plane in the opposite direction to the earth's rotation, at a distance r_2 from the center of the earth, such that $r_1 = 1.21 r_2$.

The time period of the second satellite as measured from the geostationary satellite is $\frac{24}{p}$ hours.

The value of p is _____

Ans. 2.33 (Range 2.33 to 2.33)

Sol. $\therefore T \propto r^{3/2}$

Time period of geostationary satellite is 24 hours

$$\frac{T_1}{T_2} = \frac{r_1^{3/2}}{r_2^{3/2}} \Rightarrow \frac{T_1}{T_2} = 1.331$$

Let t is time period of second w.r.t first

$$\frac{2\pi}{T_1} \cdot t = \theta \quad \dots(i)$$

$$\frac{2\pi}{T_2} \cdot t = 2\pi - \theta \quad \dots(ii)$$

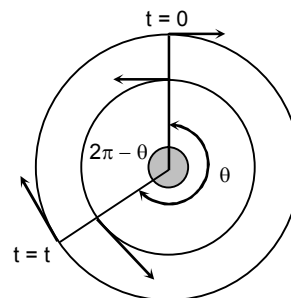
$$\Rightarrow \frac{T_1}{T_2} = \frac{2\pi - \theta}{\theta}$$

$$1.331 = \frac{2\pi}{\theta} - 1$$

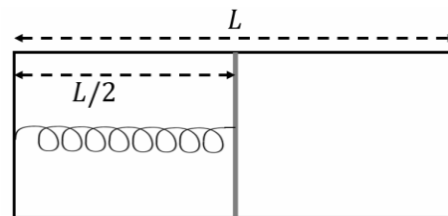
$$\Rightarrow \frac{2\pi}{\theta} = 2.331$$

$$\text{From (i) } t = T_1 \left(\frac{\theta}{2\pi} \right) = \frac{24}{2.331}$$

$$P = 2.33$$



*Q.13 The left and right compartments of a thermally isolated container of length L are separated by a thermally conducting, movable piston of area A . The left and right compartments are filled with $\frac{3}{2}$ and 1 moles of an ideal gas, respectively. In the left compartment the piston is attached by a spring with spring constant k and natural length $\frac{2L}{5}$. In thermodynamic equilibrium, the piston is at a distance



$\frac{L}{2}$ from the left and right edges of the container as shown in the figure. Under the above conditions, if the pressure in the right compartment is $P = \frac{kL}{A} \alpha$, then the value of α is ____

Ans. 0.20

Sol. In equilibrium

$$P_2 A + kx = P_1 A$$

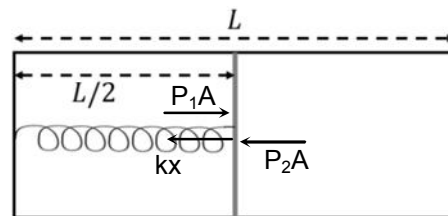
$$kx = (P_1 - P_2) A = \left(\frac{3}{2} \frac{RT}{V_0} - \frac{RT}{V_0} \right) A = \frac{RT}{2V_0} A$$

$$\therefore P_2 V_0 = 1(RT)$$

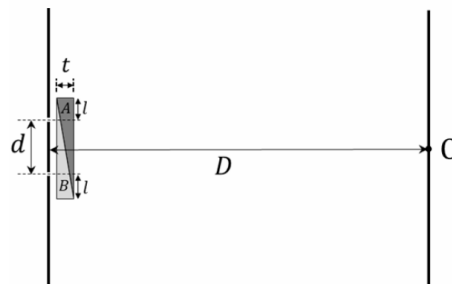
$$P_2 = \frac{RT}{V_0} = \frac{2kx}{A}$$

$$= \frac{2k}{A} [0.5L - 0.4L] = \frac{kL}{5A}$$

$$\text{Comparing } \alpha = 1/5 = 0.2$$



- Q.14 In a Young's double slit experiment, a combination of two glass wedges A and B, having refractive indices 1.7 and 1.5, respectively, are placed in front of the slits, as shown in the figure. The separation between the slits is $d = 2\text{ mm}$ and the shortest distance between the slits and the screen is $D = 2\text{ m}$. Thickness of the combination of the wedges is $t = 12\ \mu\text{m}$. The value of ℓ as shown in the figure is 1 mm . Neglect any refraction effect at the slanted interface of the wedges. Due to the combination of the wedges, the central maximum shifts (in mm) with respect to O by _____



Ans. 1.2

Sol. $\tan\theta = \frac{t_1}{\ell} = \frac{t_2}{L - \ell}$

$$L = d + 2\ell = 2 + 2 = 4\text{ mm}$$

$$\Rightarrow \frac{t_1}{1} = \frac{t_2}{3} \Rightarrow t_2 = 3t_1$$

$$\therefore t_1 + t_2 = 12\ \mu\text{m}$$

$$\Rightarrow 4t_1 = 12$$

$$\Rightarrow t_1 = 3\ \mu\text{m} \text{ and } t_2 = 9\ \mu\text{m}$$

Optical length of ray from S_1 reaching at P

$$L_1 = S_1P + t_1(n_B - 1) + t_2(n_A - 1)$$

Optical length of ray reaching at P from S_2

$$L_2 = S_2P + t_2(n_B - 1) + t_1(n_A - 1)$$

$$\Delta x = L_2 - L_1$$

$$= S_2P - S_1P + t_2(n_B - n_A) + t_1(n_A - n_B)$$

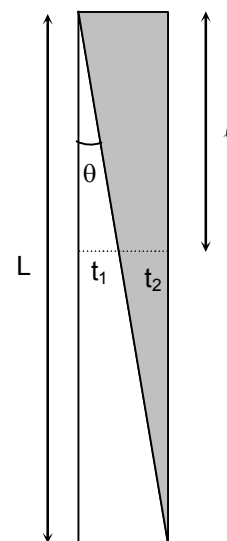
For central maxima, $\Delta x = 0$

$$0 = d\sin\theta + (n_A - n_B)(t_1 - t_2)$$

$$= d \frac{y}{D} + (1.7 - 1.5)(3 - 9) \times 10^{-6}$$

$$\Rightarrow \frac{2 \times 10^{-3}}{2} y = 0.2 \times 6 \times 10^{-6}$$

$$y = \frac{1.2 \times 10^{-6}}{10^{-3}} = 1.2 \times 10^{-3} \text{ m} = 1.2\text{ mm}$$



- *Q.15 A projectile of mass 200 g is launched in a viscous medium at an angle 60° with the horizontal, with an initial velocity of 270 m/s . It experiences a viscous drag force $\vec{F} = -c\vec{v}$ where the drag coefficient $c = 0.1\text{ kg/s}$ and \vec{v} is the instantaneous velocity of the projectile. The projectile hits a vertical wall after 2 s . Taking $e = 2.7$, the horizontal distance of the wall from the point of projection (in m) is _____

Ans. 170

Sol.

$$m \frac{d^2x}{dt^2} = -c \frac{dx}{dt}$$

$$\frac{d}{dt} \left(\frac{dx}{dt} \right) = - \frac{d}{dt} \left(\frac{c}{m} x \right)$$

$$\Rightarrow \frac{dx}{dt} = - \frac{c}{m} x + k$$

At $x = 0$, $v_x = \frac{v_0}{2}$

$$\Rightarrow \frac{dx}{dt} = \frac{v_0}{2} - \frac{c}{m} x$$

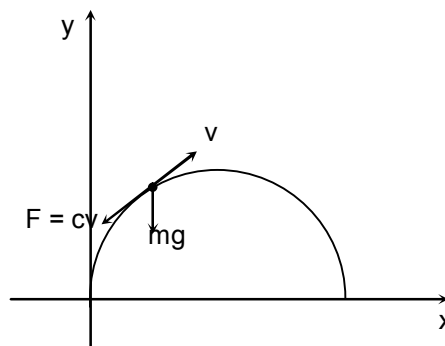
$$2 \int_0^x \frac{dx}{v_0 - x} = \int_0^t dt$$

$$2 \ln |v_0 - x| \Big|_0^x = -2$$

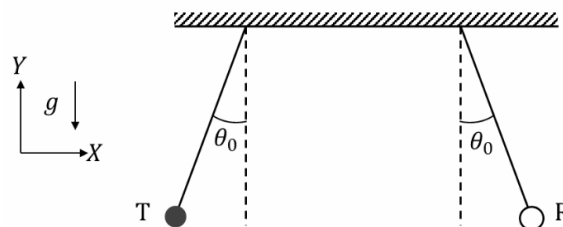
$$\Rightarrow \ln \left| \frac{v_0 - x}{v_0} \right| = -1$$

$$v_0 - x = \frac{v_0}{e}$$

$$x = v_0 - \frac{v_0}{e} = 270 - 100 = 170 \text{ m}$$



*Q.16 An audio transmitter (T) and a receiver (R) are hung vertically from two identical massless strings of length 8 m with their pivots well separated along the X axis. They are pulled from the equilibrium position in opposite directions along the X axis by a small angular amplitude $\theta_0 = \cos^{-1}(0.9)$ and



released simultaneously. If the natural frequency of the transmitter is 660 Hz and the speed of sound in air is 330 m/s, the maximum variation in the frequency (in Hz) as measured by the receiver (Take the acceleration due to gravity $g = 10 \text{ m/s}^2$) is _____

Ans. 32.01 (Range 32 to 32.02)

Sol.

$$\frac{1}{2}mv^2 = mg(L - L \cos \theta)$$

$$v = \sqrt{2gL(1 - \cos \theta)}$$

$$\Rightarrow v = \sqrt{2 \times 10 \times 8(1 - 0.9)} = \sqrt{16} = 4 \text{ m/s}$$

$$f = f_0 \left(\frac{v - v_0}{v - v_s} \right)$$

$$f_{\max} = 660 \left(\frac{330 + 4}{330 - 4} \right) = 676.20 \text{ Hz}$$

$$f_{\min} = 660 \left(\frac{330 - 4}{330 + 4} \right) = 644.19 \text{ Hz}$$

Maximum variation = $676.20 - 644.19 = 32.01 \text{ Hz}$

Chemistry

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

***Q.1** During sodium nitroprusside test of sulphide ion in an aqueous solution, one of the ligands coordinated to the metal ion is converted to

- (A) NOS^- (B) SCN^-
 (C) SNO^- (D) NCS^-

Ans. A

Sol. $\text{S}^{2-} + [\text{Fe}(\text{CN})_5\text{NO}]^{2-} \rightarrow [\text{Fe}(\text{CN})_5(\text{NOS})]^{4-}$
 (violet)

Q.2 The complete hydrolysis of ICl , ClF_3 and BrF_5 , respectively, gives

- (A) IO^- , ClO_2^- and BrO_3^- (B) IO_3^- , ClO_2^- and BrO_3^-
 (C) IO^- , ClO^- and BrO_2^- (D) IO_3^- , ClO_4^- and BrO_2^-

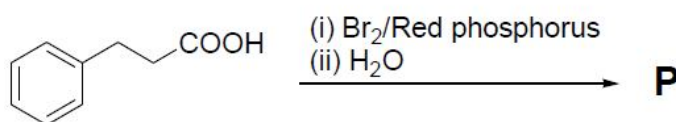
Ans. A

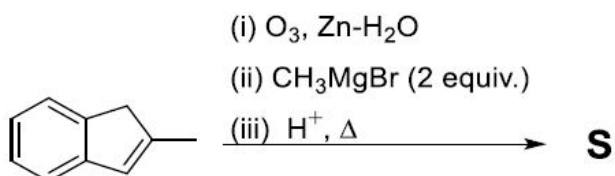
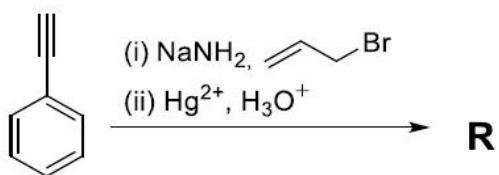
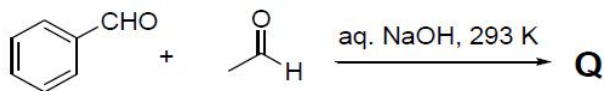
Sol. $\text{ICl} + \text{H}_2\text{O} \rightarrow \text{HOI} + \text{HCl}$
 $\hspace{10em} \searrow$
 $\hspace{10em} \rightarrow \text{H}^+ + \text{IO}^-$

$\text{ClF}_3 + 2\text{H}_2\text{O} \rightarrow \text{HClO}_2 + 3\text{HF}$
 $\hspace{10em} \searrow$
 $\hspace{10em} \rightarrow \text{H}^+ + \text{ClO}_2^-$

$\text{BrF}_5 + 3\text{H}_2\text{O} \rightarrow \text{HBrO}_3 + 5\text{HF}$
 $\hspace{10em} \searrow$
 $\hspace{10em} \rightarrow \text{H}^+ + \text{BrO}_3^-$

Q.3 Monocyclic compounds **P**, **Q**, **R** and **S** are the major products formed in the reaction sequences given below.

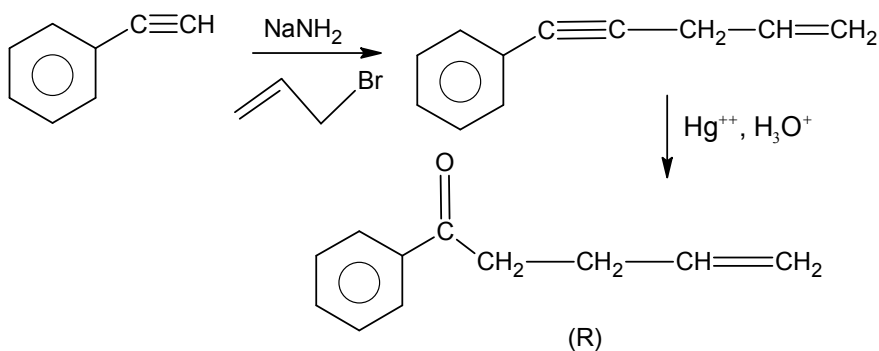
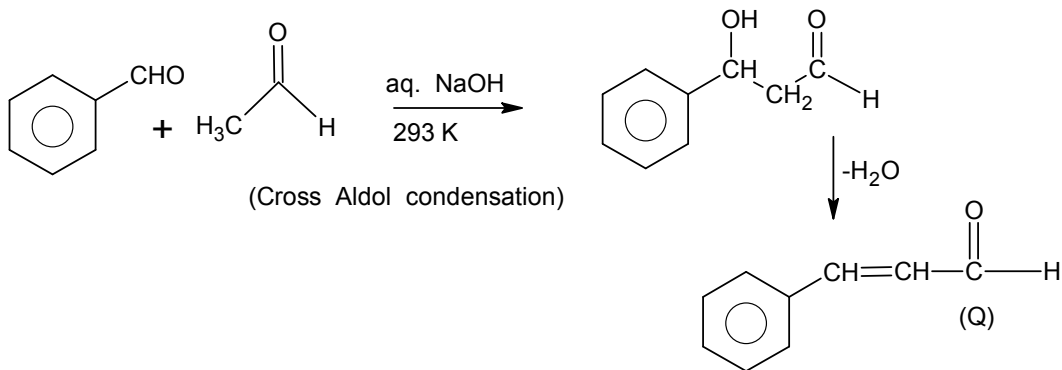
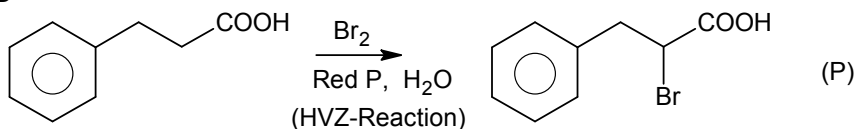


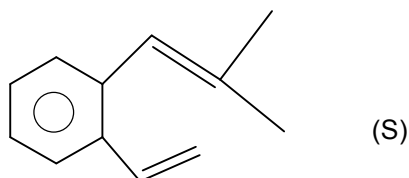
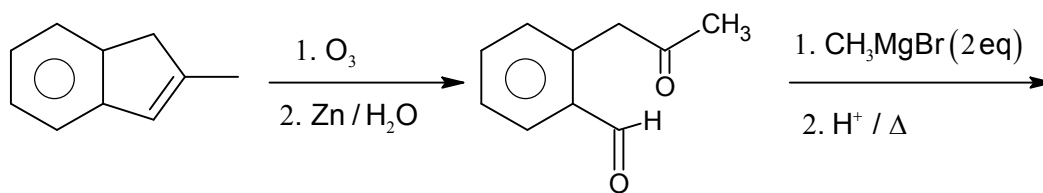


The product having the highest number of unsaturated carbon atom(s) is

- (A) P (B) Q
(C) R (D) S

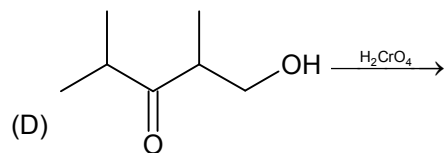
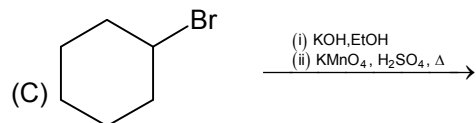
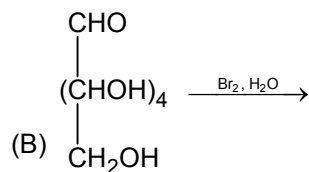
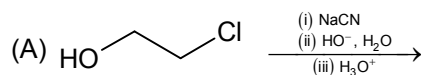
Ans. D
Sol.



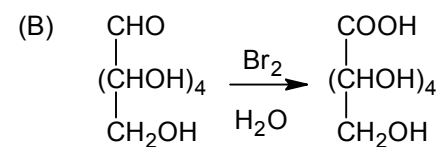
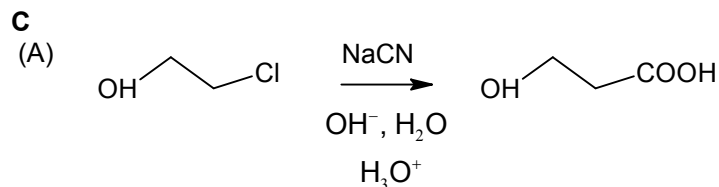


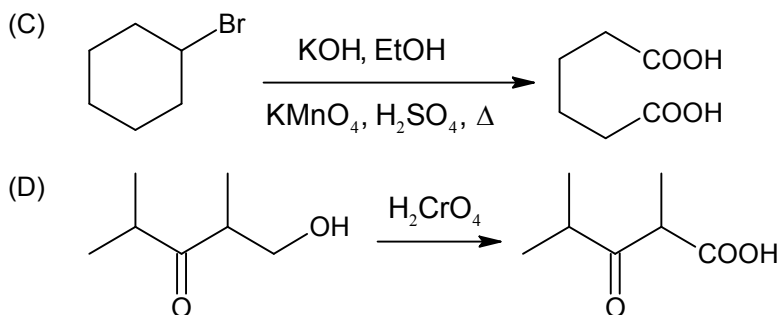
P ≡ No. of unsaturated carbon(s) = 6
 Q ≡ No. of unsaturated carbon(s) = 8
 R ≡ No. of unsaturated carbon(s) = 8
 S ≡ No. of unsaturated carbon(s) = 10

Q.4 The correct reaction/reaction sequence that would produce a dicarboxylic acid as the major product is



Ans. Sol.





SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**
Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the option is chosen (i.e. the question is unanswered);
- *Negative Marks* : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
 choosing **ONLY** (A), (B) and (D) will get +4 marks;
 choosing **ONLY** (A) and (B) will get +2 marks;
 choosing **ONLY** (A) and (D) will get +2 marks;
 choosing **ONLY** (B) and (D) will get +2 marks;
 choosing **ONLY** (A) will get +1 mark;
 choosing **ONLY** (B) will get +1 mark;
 choosing **ONLY** (D) will get +1 mark;
 choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
 choosing any other combination of option(s) will get -2 marks.

- *Q.5 The correct statement(s) about intermolecular forces is(are)
- (A) The potential energy between two point charges approaches zero more rapidly than the potential energy between a point dipole and a point charge as the distance between them approaches infinity.
- (B) The average potential energy of two rotating polar molecules that are separated by a distance r has $1/r^3$ dependence.
- (C) The dipole-induced dipole average interaction energy is independent of temperature.
- (D) Nonpolar molecules attract one another even though neither has a permanent dipole moment.

Ans. **D or C, D**

Sol. (A) Ion - Ion $\propto \frac{1}{r}$

Ion - dipole $\propto \frac{1}{r^2}$

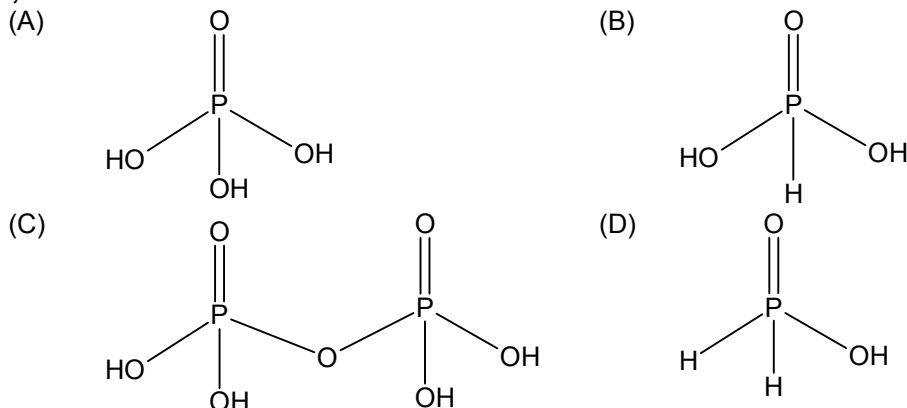
- (B) $F \propto \frac{1}{r^6}$ (For rotating polar molecules)
 (C) As $T \uparrow \Rightarrow$ Energy \downarrow
 (D) London forces or dispersion forces are the force of attraction between the non-polar molecules.

Q.6 The compound(s) with P – H bond(s) is(are)

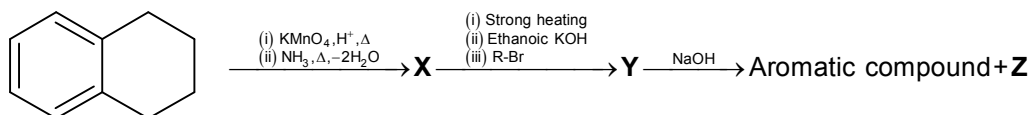
- (A) H_3PO_4 (B) H_3PO_3
 (C) $H_4P_2O_7$ (D) H_3PO_2

Ans. B, D

Sol.



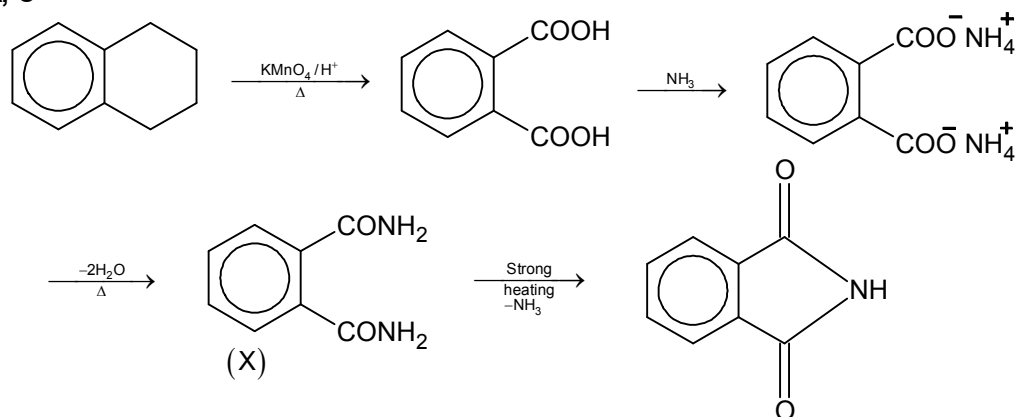
Q.7 For the reaction sequence given below, the correct statement(s) is(are)

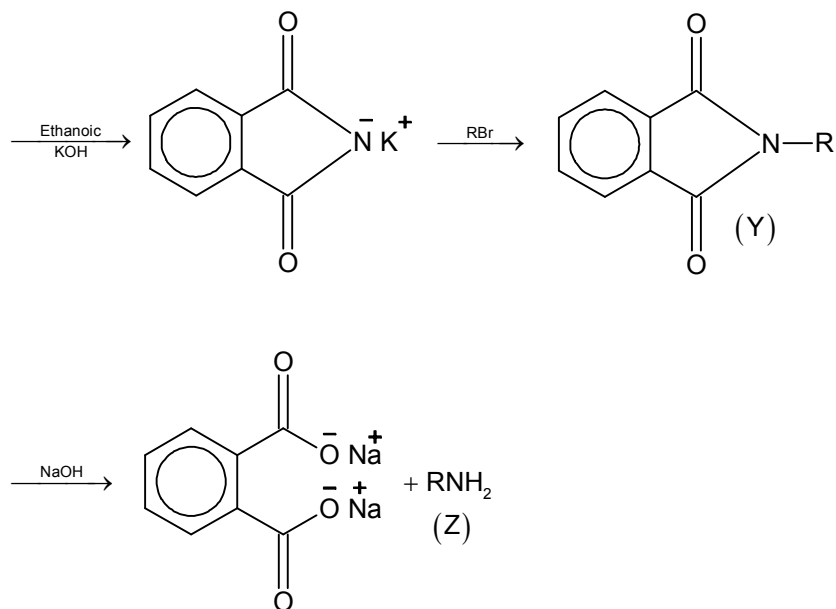


- (A) Both **X** and **Y** are oxygen containing compounds.
 (B) **Y** on heating with $CHCl_3/KOH$ forms isocyanide.
 (C) **Z** reacts with Hinsberg's reagent.
 (D) **Z** is an aromatic primary amine.

Ans. A, C

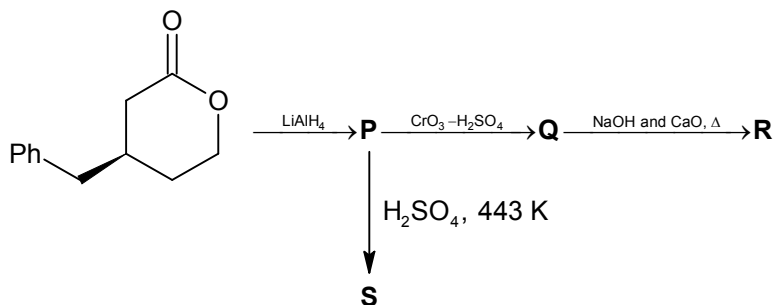
Sol.





(X, Y) – Oxygen containing compounds.
 (Z) – 1° amine, so reacts with Hinsberg's reagent.

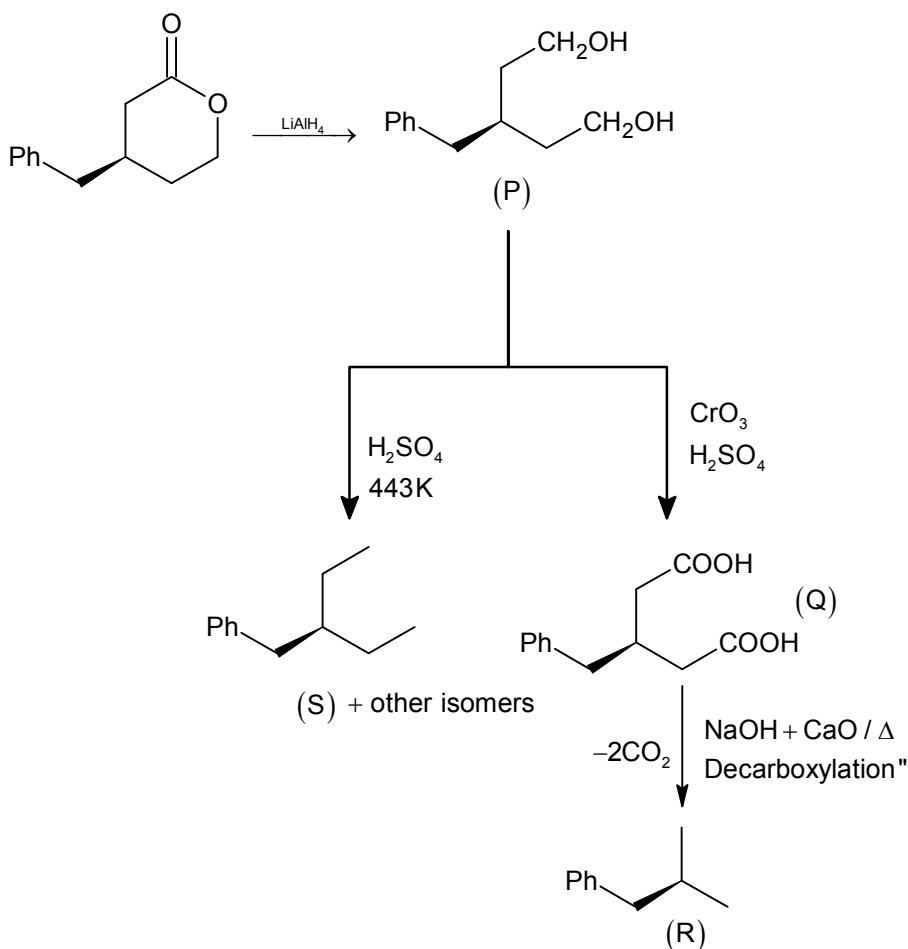
Q.8 For the reaction sequence given below, the correct statement(s) is(are)



- (A) P is optically active.
- (B) S gives Bayer's test.
- (C) Q gives effervescence with aq. NaHCO_3 .
- (D) R is an alkyne.

Ans. B, C

Sol.



- P is optically inactive.
- S gives Bayer's test
- Q gives effervescences with NaHCO_3 (aq)
- R is an Alkane.

SECTION 3 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks : 0 In all other cases.

Q. 9 The density (in g cm^{-3}) of the metal which forms a cubic close packed (ccp) lattice with an axial distance (edge length) equal to 400 pm is _____.
Use: Atomic mass of metal = 105.6 amu and Avogadro's constant = $6 \times 10^{23} \text{ mol}^{-1}$

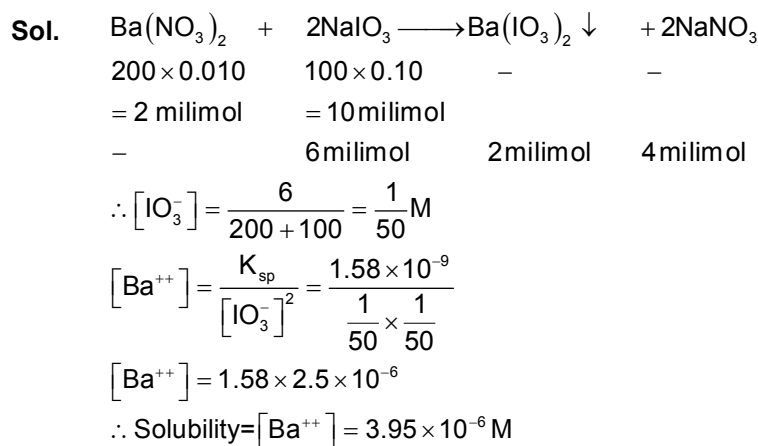
Ans. 11.00

Sol. For ccp, $Z = 4$, $M = 105.6 \text{ g mol}^{-1}$
 $N_A = 6 \times 10^{23} \text{ mol}^{-1}$
 $a = 400 \text{ pm} = 4 \times 10^{-8} \text{ cm}$
 $\therefore d = \frac{ZM}{a^3 N_A} = \frac{4 \times 105.6}{(4 \times 10^{-8})^3 \times 6 \times 10^{23}}$
 $d = \frac{4 \times 105.6}{64 \times 6 \times 10^{-24} \times 10^{23}}$
 $d = \frac{1056}{96} = 11 \text{ cm}^{-3}$

***Q. 10** The solubility of barium iodate in an aqueous solution prepared by mixing 200 mL of 0.010 M barium nitrate with 100 mL of 0.10 M sodium iodate is $X \times 10^{-6} \text{ mol dm}^{-3}$. The value of X is _____.

Use: Solubility product constant (K_{sp}) of barium iodate = 1.58×10^{-9} .

Ans. 3.95



Q. 11 Adsorption of phenol from its aqueous solution on to fly ash obeys Freundlich isotherm. At a given temperature, from 10 mg g^{-1} and 16 mg g^{-1} aqueous phenol solutions, the concentrations of adsorbed phenol are measured to be 4 mg g^{-1} and 10 mg g^{-1} , respectively. At this temperature, the concentration (in mg g^{-1}) of adsorbed phenol from 20 mg g^{-1} aqueous solution of phenol will be _____.

Use: $\log_{10} 2 = 0.3$

Ans. 16.00

Sol. $\frac{x}{m} = KC^{1/n}$

$C = 10 \text{ mg g}^{-1} \Rightarrow \frac{x}{m} = 4 \text{ mg g}^{-1}$

$C = 16 \text{ mg g}^{-1} \Rightarrow \frac{x}{m} = 10 \text{ mg g}^{-1}$

$\Rightarrow 4 = K(10)^{1/n} \quad (1)$

$10 = K(16)^{1/n} \quad (2)$

$\Rightarrow \frac{4}{10} = \left(\frac{10}{16}\right)^{1/n}$

$$\Rightarrow \log 4 - \log 10 = \frac{1}{n} [\log 10 - \log 16]$$

$$\Rightarrow n = \frac{1 - 4 \times .30}{2 \times .30 - 1} = \frac{0.2}{0.4} \Rightarrow n = \frac{1}{2}$$

$$\text{Now } \frac{x}{m} = K(20)^{\frac{1}{x}} \quad (3)$$

Dividing (3) by (1)

$$\frac{x/m}{4} = \left(\frac{20}{10}\right)^{\frac{1}{x}} = 4$$

$$\frac{x}{m} = 16 \text{ mg g}^{-1}$$

- Q. 12** Consider a reaction $A + R \longrightarrow \text{Product}$. The rate of this reaction is measured to be $k[A][R]$. At the start of the reaction, the concentration of R, $[R]_0$, is 10-times the concentration of A, $[A]_0$. The reaction can be considered to be a pseudo first order reaction with assumption that $k[R] = k'$ is constant. Due to this assumption, the relative error (in %) in the rate when this reaction is 40% complete, is _____.
[k and k' represent corresponding rate constants]

Ans. 4.17

- Sol.** Let $[A]_0 = 1 \text{ M}$, $[R]_0 = 10 \text{ M}$
Considering pseudo first order reaction
 $r = k'[A]$, on 40% completion
or $r_1 = k[10][0.6] \Rightarrow r_1 = 6K \quad \dots (1)$
Considering second order reaction
 $r = k[R][A]$, on 40% completion
 $r_2 = k[9.6][0.6] \Rightarrow r_2 = 5.76 K \quad \dots (2)$
 \therefore From (1) and (2)
Relative error = $\frac{6K - 5.76K}{5.76K} \times 100$
 $= \frac{.24}{5.76} \times 100 = \frac{100}{24} = 4.166$
 $\cong 4.17\%$

- Q. 13** At 300 K, an ideal dilute solution of a macromolecule exerts osmotic pressure that is expressed in terms of the height (h) of the solution (density = 1.00 g cm^{-3}) where h is equal to 2.00 cm. If the concentration of the dilute solution of the macromolecule is 2.00 g dm^{-3} , the molar mass of the macromolecule is calculated to be $X \times 10^4 \text{ g mol}^{-1}$. The value of X is _____.
Use: Universal gas constant (R) = $8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ and acceleration due to gravity (g) = 10 m s^{-2} .

Ans. 2.49

- Sol.** Osmotic pressure $\Pi = h \rho g$
 $= 2 \times 10^{-2} \text{ m} \times \frac{1 \text{ kg}}{\text{m}^3} \times \frac{10 \text{ m}}{\text{s}^2}$
 $= 0.2 \text{ Pa}$

Now $0.2 = CRT$

$$\Rightarrow 0.2 = 8.3 \times 300 \times \frac{2}{M}$$

$$\Rightarrow M = 8.3 \times 3000$$

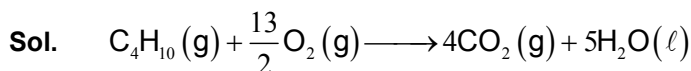
$$\Rightarrow M = 24900.0 = 2.49 \times 10^4 \text{ g mol}^{-1}$$

Q. 14 An electrochemical cell is fueled by the combustion of butane at 1 bar and 298 K. Its cell potential is $\frac{X}{F} \times 10^3$ volts, where F is the Faraday constant. The value of X is _____.

Use: Standard Gibbs energies of formation at 298 K are :

$$\Delta_f G_{\text{CO}_2}^\circ = -394 \text{ kJ mol}^{-1}; \Delta_f G_{\text{water}}^\circ = -237 \text{ kJ mol}^{-1}; \Delta_f G_{\text{butane}}^\circ = -18 \text{ kJ mol}^{-1}$$

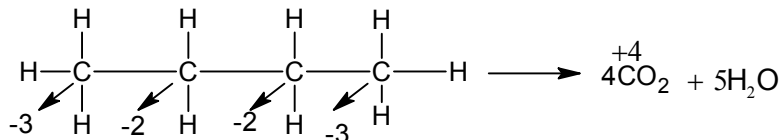
Ans. 105.50



$$\Delta_r G^\circ = 4 \times (-394) + 5 \times (-237) - (1 \times -18)$$

$$\Delta_r G^\circ = -1576 - 1185 + 18$$

$$\Delta_r G^\circ = -1558 - 1185 = -2743 \text{ KJ mol}^{-1}$$



No. of e^- (s) exchanged (lost)

$$= 2 \times [4 - (-3)] + 2 \times [4 - (-2)]$$

$$= 14 + 12 = 26$$

$$\Delta G = -nF E_{\text{cell}}$$

$$-2743 \times 10^3 = -26 \times F \times E_{\text{cell}}$$

$$E_{\text{cell}} = \frac{2743}{26F} \times 10^3 = \frac{105.5}{F} \times 10^3$$

X = 105.50

Q. 15 The sum of the spin only magnetic moment values (in B.M.) of $[\text{Mn}(\text{Br})_6]^{3-}$ and $[\text{Mn}(\text{CN})_6]^{3-}$ is _____.

Ans. 7.72

Sol. $[\text{MnBr}_6]^{3-} \Rightarrow \text{Mn}^{3+} \Rightarrow 3d^4 \Rightarrow n = 4$, since Br^- is a weak field ligand.

$[\text{Mn}(\text{CN})_6]^{3-} \Rightarrow \text{Mn}^{3+} \Rightarrow 3d^4 \Rightarrow n = 2$, since CN^- is a strong field ligand.

$$\therefore \mu_s(\text{total}) = \sqrt{4(4+2)} + \sqrt{2(2+2)}$$

$$= \sqrt{24} + \sqrt{8}$$

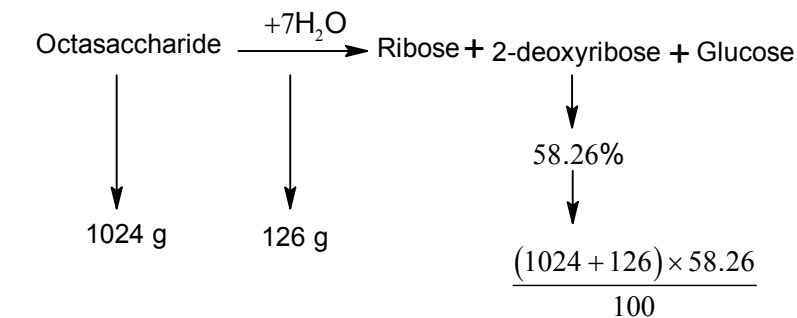
$$= 4.90 + 2.82$$

$$\therefore \mu_s(\text{total}) = 7.72 \text{ BM}$$

- Q. 16** A linear octasaccharide (molar mass = 1024 g mol^{-1}) on complete hydrolysis produces three monosaccharides: ribose, 2-deoxyribose and glucose. The amount of 2-deoxyribose formed is 58.26% (w/w) of the total amount of the monosaccharides produced in the hydrolyzed products. The number of ribose unit(s) present in one molecule of octasaccharide is _____.
- Use:** Molar mass (in g mol^{-1}): ribose = 150, 2-deoxyribose = 134, glucose = 180; Atomic mass (in amu) : H = 1, O = 16

Ans. 2.00

Sol.



= 670 g i.e. 5 units of 2-deoxyribose.

$$\therefore \text{mass of Ribose} + \text{mass of Glucose} = 1150 - 670 = 480 \text{ g}$$

$$\begin{array}{ccc}
 \downarrow & & \downarrow \\
 150x & & 180y
 \end{array}$$

$$\text{i.e. } 150x + 180y = 480$$

$$\Rightarrow x = 2, y = 1$$

\therefore Number of ribose unit (s) = 2