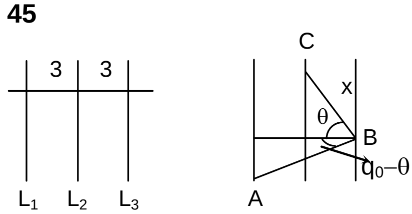
FIITJEE

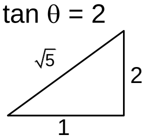
Mathematics IOQM Solutions (2021-22)

PART A-(RMO 2021-22)

1. **45**



$x \cos\theta = 3$
$x \sin\theta = 6$
$\tan\theta = 2$



$\Rightarrow \sin\theta = \dfrac{2}{\sqrt{5}}$

$x\left(\dfrac{2}{\sqrt{5}}\right) = 6$

$x = 3\sqrt{5}$
$x^2 = 45$
Area = 45

2. **68**
1, 2, 3 – 101
1 – 33 blue
34 to 101 red
ANS = 68 numbers by red pen

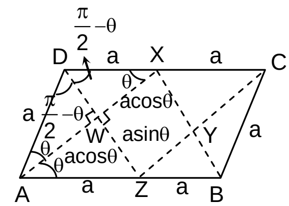
3. **8**
Sides = 3, 5, 7 $\Rightarrow$ L = 15

$\cos\theta = \dfrac{9+25-49}{2\times3\times5} = \dfrac{-16}{2\times35} = \dfrac{-8}{15}$; $\dfrac{a}{L} = \dfrac{120^\circ}{15} = 8$

4. **98**
Assuming a, b, c are non-zero
Sum of all numbers = $3^5(a + b + c) \times 111, 111$
= 593999406
a + b + c = 22
Possible combination include when two of the digits are 8 and 9
$\therefore$ Largest possible remainder is 98 in all situations (even if one of them is a zero)

5. **40**

We can clearly



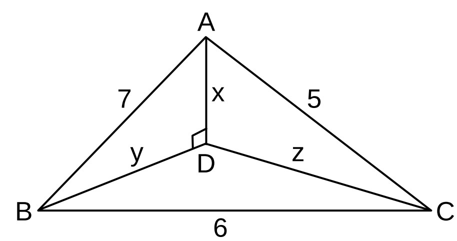
see that XYZW as area $a^2\sin\theta\cos\theta = 10$

$\Rightarrow$ Area of ABCD = $2a.a\sin 2\theta$

$= 4a^2\sin\theta\cos\theta = 40$

6. **30**

Assume ΔABC with sides AB = 7, BC = 6, CA = 5. D is a point which is at a distance x, y, z units from A, B, C respectively.



In ΔADB

$x^2 + y^2 = 49$ (Given)

$\Rightarrow \angle ADB = 90°$

In ΔBDC

$y^2 + yz + z^2 = 36$

$\Rightarrow \frac{y^2+z^2-36}{2yz} = -\frac{1}{2} \Rightarrow \cos(\angle BDC) = -\frac{1}{2}$

$\Rightarrow \angle BDC = 120°$

Similarly, $\angle ADC = 150°$

Now, Area of ΔABC = Area of ΔADB + Area of ΔBDC + Area of ΔADC

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}xy + \frac{1}{2}yz\sin(120°) + \frac{1}{2}zx\sin(150°)$

$6\sqrt{6} = \frac{2xy+\sqrt{3}yz+zx}{4} \Rightarrow 2xy+\sqrt{3}yz+zx = 24\sqrt{6} = p\sqrt{q}$

$\Rightarrow p + q = 30$

7. **35**

Case I : $f(1) = f(2) = f(3) \rightarrow 5$

Case II : $f(1) = f(2) < f(3) \rightarrow {}^5C_2$

or $f(1) < f(2) = f(3) \rightarrow {}^5C_2$

Case III : $f(1) < f(2) < f(3) \rightarrow {}^5C_3$

Total = 35

8. **24**

$4 \le \left[\sqrt{\left[\sqrt{\left[\sqrt{N}\right]}\right]}\right] < 5 \Rightarrow 16 \le \left[\sqrt{\left[\sqrt{N}\right]}\right] < 25$

$\Rightarrow 16 \le \sqrt{\left[\sqrt{N}\right]} < 25 \Rightarrow 256 \le \sqrt{N} < 625$

$\Rightarrow (256)^2 \le N < (625)^2$; $N = (625)^2 - 1 = 390624$. Sum of digits = 24

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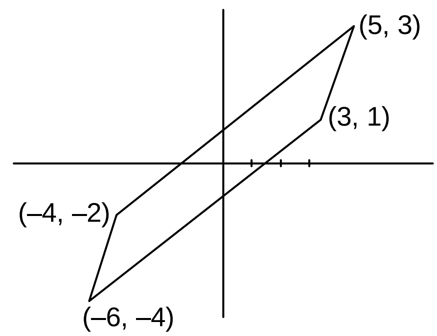
9. **8**

Let A_n = Area of quad (P_n, P_{n+1}, P_{n+1}, P_{n+3}) = $\frac{1}{2}\begin{vmatrix} x_n & y_n \\ x_{n+1} & y_{n+1} \\ x_{n+2} & y_{n+2} \\ x_{n+3} & y_{n+3} \end{vmatrix}$

$$A_{n+1} = \frac{1}{2}\begin{vmatrix} -\left(\frac{3x_n - y_n}{2}\right) & -\left(\frac{x_n + y_n}{2}\right) \\ ..... & ..... \\ ..... & ..... \end{vmatrix} = \frac{1}{2x^n}\begin{vmatrix} 4x_n & x_n + y_n \\ ..... & ..... \\ ..... & ..... \end{vmatrix}$$

$$= \frac{1}{2}\times\frac{4}{4}\begin{vmatrix} x_n & x_n + y_n \\ ..... & ..... \\ ..... & ..... \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_n & y_n \\ ..... & ..... \\ ..... & ..... \end{vmatrix} \Rightarrow A_n = A_{n+1} \Rightarrow A_0 = A_1 = A_2 = A_3 \; .....$$

$P_0 = (3, 1)$, $P_1 = (-4, -2)$; $P_2 = \phi(5, 3)$, $P_3 \equiv (-6, -4)$



$$= \frac{1}{2}\begin{vmatrix} 3 & 1 \\ 5 & 3 \\ -4 & -2 \\ -6 & -4 \\ 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[(9-5) + (-10+12) + (16-12) + (-6+12)\right]$$

$$= \frac{1}{2}\left[4+2+4+6\right] = 8$$

10. **40**

$P\left(\sqrt{7}+\sqrt{5}\right) = 2\left(\sqrt{7}-\sqrt{5}\right) \Rightarrow P\left(\sqrt{7}+\sqrt{5}\right) = \left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)^2$

Let $\sqrt{7}+\sqrt{5} = x \Rightarrow \left(\sqrt{7}-\sqrt{5}\right)^2 = 24 - x^2 \Rightarrow P(x) = x\left(24 - x^2\right) \Rightarrow P(2) = 2 \times 20 = 40$

11. **12**

Let husbands be arranged in 3! ways. Then their wives can be arranged in only 2 ways
Total = 3! × 2 = 12

12. **73**

If N rooks are placed on any N unit squares and we want to select 7 rooks such that they are non-attacking, then we must select 7 unit squares from 7 different rows and 7 different columns.
⇒ For minimum value of N we must fill 73 unit squares so that we get atleast one rook from 7 different rows and 7 different columns.
Because if I take N = 72, then these 72 rooks can be placed in 12 rows and 6 columns. Hence we can't select 7 rooks from 7 different columns.
⇒ 73

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