

FIITJEE

Mathematics IOQM Questions (2021-22)

PART – B-INMO

Time: 2.5 hours

Instructions:

- Calculators (in any form) and protractors are not allowed.
- Rulers and compasses are allowed.
- All questions carry equal marks. Maximum marks : 51
- No marks will be awarded for stating an answer without justification.
- Answer all the questions.
- PLEASE READ THE INSTRUCTIONS ON THE ANSWER BOOKLET VERY CAREFULLY BEFORE ANSWERING THE QUESTIONS.

1. Let D be an interior point on the side BC of an acute-angled triangle ABC . Let the circumcircle of triangle ADB intersect AC again at $E (\neq A)$ and the circumcircle of triangle ADC intersect AB again at $F (\neq A)$. Let AD , BE and CF intersect the circumcircle of triangle ABC again at $D_1 (\neq A)$, $E_1 (\neq B)$ and $F_1 (\neq C)$, respectively. Let I and I_1 be the incentres of triangles DEF and $D_1E_1F_1$, respectively. Prove that E, F, I, I_1 are concyclic.

2. Find all natural numbers n for which there exists a permutation σ of $1, 2, \dots, n$ such that

$$\sum_{i=1}^n \sigma(i)(-2)^{i-1} = 0.$$

Note: A permutation of $1, 2, \dots, n$ is a bijective function $\{1, 2, \dots, n\}$ to itself.

3. For a positive integer N , let $T(N)$ denote the number of arrangements of the integers $1, 2, \dots, N$ into a sequence a_1, a_2, \dots, a_N such that $a_i > a_{2i}$ for all i , $1 \leq i < 2i \leq N$ and $a_i > a_{2i+1}$, for all i , $1 \leq 2i + 1 \leq N$. For example, $T(3)$ is 2, since the possible arrangements are 321 and 312.

(a) Find $T(7)$.

(b) If K is the largest non-negative integer so that 2^K divides $T(2^n - 1)$, show that $K = 2^n - n - 1$.

(c) Find the largest non-negative integer K so that 2^K divides $T(2^n + 1)$.
