

Pre-RMO 2019

FIITJEE ANSWER KEYS

1.	4	2.	13	3.	13	4.	36
5.	10	6.	29	7.	51	8.	49
9.	14	10.	55	11.	6	12.	18
13.	10	14.	53	15.	45	16.	40
17.	30	18.	20	19.	13	20.	Bonus
21.	17	22.	78	23.	55	24.	37
25.	48	26.	50	27.	84	28.	15
29.	47	30.	64				



FIITJEE SOLUTIONS Pre-RMO 2019

1. $2x + x\sqrt{2} = 5$

$$x = \frac{5}{2 + \sqrt{2}} \Rightarrow x\sqrt{2} = \frac{5}{\sqrt{2} + 1}$$

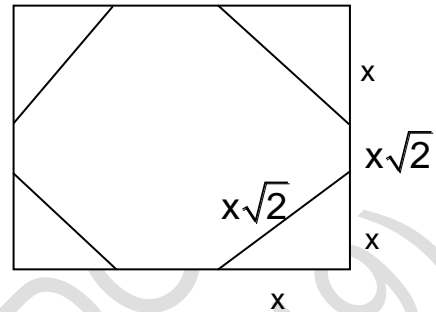
$$A = 2(1 + \sqrt{2})s^2 = 2(1 + \sqrt{2}) \cdot \frac{25}{(\sqrt{2} + 1)^2}$$

$$= 50(\sqrt{2} - 1)$$

$$= 20.70$$

$$\text{Area removed} = 25 - 20.70 = 4.3$$

Hence answer is 4.



2. $\left(x + \frac{1}{x}\right)^2 + a\left(x + \frac{1}{x}\right) + b = x^2 + ax + b + \frac{1}{x^2} + \frac{a}{x} + b$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + b \Rightarrow b = 2$$

$$f(x) = x^2 + ax + 2$$

Since roots are integers.

$$D = a^2 - 8,$$

$$\text{must be perfect square} \Rightarrow a^2 = 9$$

$$\Rightarrow a^2 + b^2 = 9 + 4 = 13$$

3. $x_5 = 1 + x_1x_2x_3x_4 \Rightarrow x_1x_2x_3x_4 = 42$

$$x_6 = 1 + x_1x_2x_3x_4x_5 = 1 + 42 \times 43 = 1807 = 13 \times 139$$

$$\Rightarrow \text{Largest prime factor} = 139$$

$$\Rightarrow \text{Required sum} = 13$$

4.

The situation clearly depicts a 9 – point star.

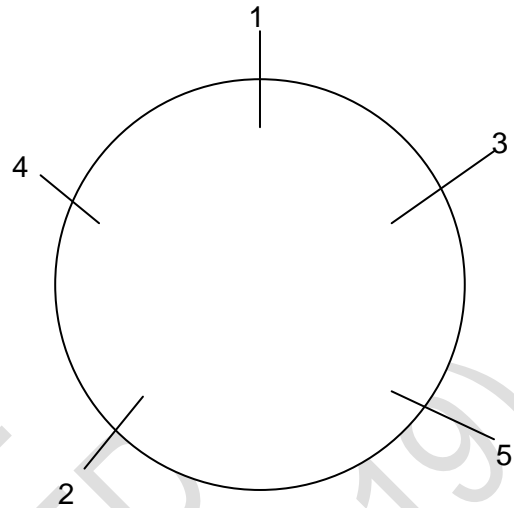
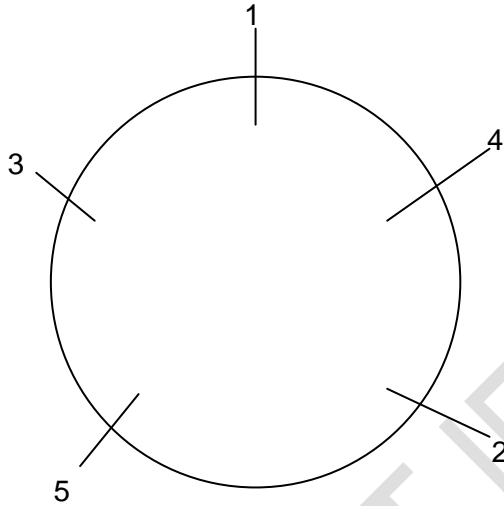
$$\Rightarrow \text{Distance traveled} = 4 \times 9 = 36 \text{ feet}$$



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5. Answer is $2 \times 5 = 10$ as can be seen below



6. $\overline{abc} = 100a + 10b + c$
 $a^2 + b^2 = c^2$
 a, b, c must be single digit numbers
 $\Rightarrow a = 4; b = 3; c = 5$ is the only possibility
 $\overline{abc} = 435$
 \Rightarrow Largest prime factor = 29

7. 2 times between 12 and 1, the arms make an angle of 90 degrees. At

$$\frac{180}{11} \text{ and at } \frac{540}{11} \text{ minutes}$$

$$\therefore \text{So difference is} = \frac{360}{11} = 32 + \frac{8}{11}$$

$$\therefore a + b + c = 51$$

Answer is 51

8. $x^{2^n} + x + 1$ should be divisible by ω and ω^2
 $n = 3, 5, 7, \dots, 99$

Satisfy

$$\text{So number of values of } x = \frac{100}{2} - 1 = 49$$

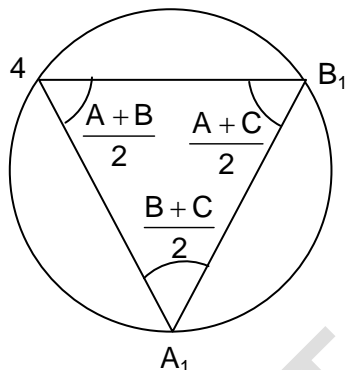


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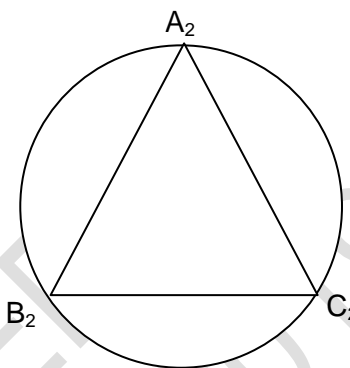
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9. $\frac{311}{99}$
 $p = 311; q = 99 \Rightarrow p - 3q = 311 - 297 = 14$

10.



\therefore Smallest angle
 $\therefore \frac{180 + 40}{4} = 55^\circ$



$$A_2 = \frac{\frac{A+B}{2} + \frac{A+C}{2}}{2} = \frac{180 + A}{4}$$

$$B_2 = 180 + \frac{B}{4}$$

$$C_2 = 180 + \frac{C}{4}$$

11. $\sin kC = \frac{1 - \cos A \cos B}{\sin A \sin B} \leq 1 \Rightarrow 1 \leq \cos A \cos B + \sin A \sin B$
 $\Rightarrow \cos(A - B) \geq 1 \Rightarrow \cos(A - B) = 1 \Rightarrow A = B \Rightarrow \sin kC = 1$
 $\Rightarrow kC = 90^\circ \Rightarrow C = \frac{90^\circ}{k}$
 $k = 1, 3, 5, 9, 15, 45$
 Number of integer values = 6

12. The equation $\frac{1}{\sqrt{a_1}} + \frac{1}{\sqrt{a_2}} = 1$ has no solution for distinct a_1 and a_2
 Now, $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$ i.e. $\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{9}} + \frac{1}{\sqrt{36}} = 1$
 Therefore 3 is a good number but 2 is not.



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Now if k is a good number, then $k+1$ is also good because if

$$\frac{1}{\sqrt{a_1}} + \frac{1}{\sqrt{a_2}} + \frac{1}{\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_k}} = 1$$

$$\text{then } \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{4a_1}} + \frac{1}{\sqrt{4a_2}} + \frac{1}{\sqrt{4a_3}} + \dots + \frac{1}{\sqrt{4a_k}} = 1$$

$$\text{Hence } f(n) = 3 + 4 + \dots + (n+2) = \frac{n(n+5)}{2}$$

$$\text{If } f(n) | f(n+5)$$

$$n | n+10$$

$$n | 10$$

$$\therefore n = 1, 2, 5, 10$$

Hence required sum is 18

$$13. \quad (x_1 + x_2 + \dots + x_{101})^2 = 101 + 2S$$

$$2S = (x_1 + x_2 + \dots + x_{101})^2 - 101$$

$$S = \frac{1}{2} \left[(x_1 + x_2 + \dots + x_{101})^2 - 101 \right] \geq 0$$

$$(x_1 - x_2 + \dots - x_{101})^2 \geq 101$$

$$|x_1 + x_2 + \dots + x_{101}| \geq 10 \dots$$

$$\Rightarrow S_{\min} = \frac{1}{2} [11^2 - 101] = 10$$

$$14. \quad 9N + 7 = P^2$$

$$\text{Or } N = \frac{P^2 - 7}{9}$$

$$\text{For } P = 22 \quad N = 53$$

Answer is 53

15. Let (x, y) be the pair of number of vertices of a diagonal.

If (x, y) is of type

even – even \rightarrow 3 parallel diagonals

odd – odd \rightarrow 3 parallel diagonals

odd – even \rightarrow 2 parallel diagonals



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$$\begin{aligned} \text{Pairs of parallel diagonals} &= \frac{2\binom{5}{2} \times 3 + 5 \times 3 \times 2}{2} \\ &= \frac{60 + 30}{2} = 45 \end{aligned}$$

16. $13x + 17y = 10000$

$$x = 17n + 16$$

$$\left. \begin{aligned} x &= 17n + 16 \\ y &= 576 - 13n \end{aligned} \right\} \text{Use Bezout's identity}$$

$$x - y = 10(3n - 56)$$

Since $|x - y|$ should be min. $m \Rightarrow n = 19$

$$\Rightarrow x = 339; y = 329$$

\therefore In next year school paid a total of $17x + 13y = 10040$

Thus school paid Rs. 40/- more.

17. $C = 1 \rightarrow b = 4 \rightarrow a \rightarrow 2$ choices

$$b = 3 \rightarrow a \rightarrow 4$$
 choices

$$b = 2 \rightarrow a \rightarrow 5$$
 choices

$$b = 1 \rightarrow a \rightarrow 7$$
 choices

18 triplets

$$C = 2 \rightarrow b = 3 \rightarrow a \rightarrow 1$$
 choice

$$b = 2 \rightarrow a \rightarrow 3$$
 choice

$$b = 1 \rightarrow a \rightarrow 5$$
 choice

9 triplets

$$C = 3 \rightarrow b = 2 \rightarrow a \rightarrow 1$$
 choice

$$b = 1 \rightarrow a \rightarrow 2$$
 choices

3 triplets

Possible triplets = $18 + 9 + 3 = 30$



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18. Let $a = Hx$ $x < y$
 $b = Hy$ $H = \text{H. C. F. of } a, b$
 L.C.M = Hxy
 Now $495H = HXY$
 $XY = 495$
 $9(55) \quad H = 12, 13, \dots, 18 \rightarrow 7 \text{ values}$

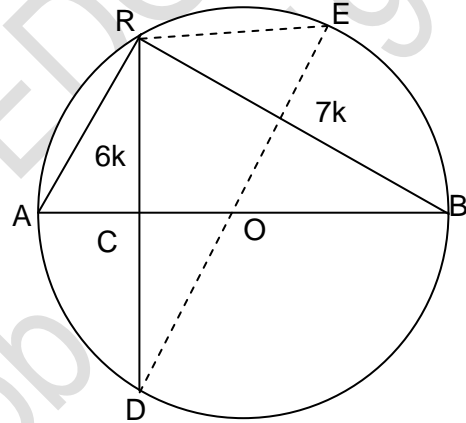
 $11(45) \quad H = 10, 11, 12, \dots, 22 \rightarrow 13 \text{ values}$

$\therefore \text{Total values} = 7 + 13 = 20$

19. $CD = \sqrt{42}k, DE = 13k \Rightarrow RE = k$

$$\frac{[ABD]}{[CDE]} = \frac{\frac{1}{2} \times AB \times CD}{\frac{1}{2} \times CD \times RE} = \frac{AB}{RE} = \frac{13k}{k} = 13$$

Ans. 13



20. **Bonus as Question is wrong.**

21. $E = \{a, b, c, d, e\}$

A and B are of type $\{a\}, \{b, c, d, e\}$

OR

A and B are of type $\{a, b\}, \{c, d, e\}$

$ab + cde$ should be odd which can be obtained when a and b are 5 and 7 respectively and c, d, e are 6, 8, 9 respectively.

$\Rightarrow 35 + 432 = 467$

$\Rightarrow \text{Answer } 4 + 6 + 7 = 17$

22. $\sqrt{n-1} < \sqrt{n} < \sqrt{n+1} \Rightarrow \sqrt{n+1} - \sqrt{n} < \frac{1}{2\sqrt{n}} < \sqrt{n} - \sqrt{n-1}$

$\Rightarrow \sqrt{1600} - 1 < \sum_{n=1}^{1599} \frac{1}{2\sqrt{n}} < \sqrt{1599}$



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$$\Rightarrow 78 < \sum_{n=1}^{1599} \frac{1}{2\sqrt{n}} < 79.97$$

Answer = 78

23. For the sum PA+PB+PC+PD to be minimum, P must be the point of intersection of diagonals. Moreover since it is a cyclic quadrilateral we have by Power of Point theorem,
 $DP \times PB = AP \times PC \Rightarrow 6 \times 4 = 3 \times 8 = 24$.

So ABCD is cyclic

$$\text{Area of quadrilateral} = \Delta APB + \Delta BPC + \Delta CPD + \Delta PDA = \frac{1}{2}(3 \times 4 + 4 \times 8 + 8 \times 6 + 6 \times 3) = 55$$

24. $f(n) = {}^n C_0 2^n + {}^n C_2 2^{n-2} + {}^n C_4 2^{n-4} \dots$
 $= \frac{(2+1)^n + (2-1)^n}{2}$

$$f(n) = \frac{3^n + 1}{2}$$

$$\frac{f(9)}{f(3)} = \frac{3^9 + 1}{3^3 + 1} = 703$$

Prime factors of 703 = 19 and 37
 Largest prime factor = 37

25. $TB \cdot TD = TD^2$
 $16(16 + d) = y^2 \dots\dots(1)$

Also, by similarity

$$\frac{16+r}{48+y} = \frac{r}{48}$$

$$ry = 48 \times 16$$

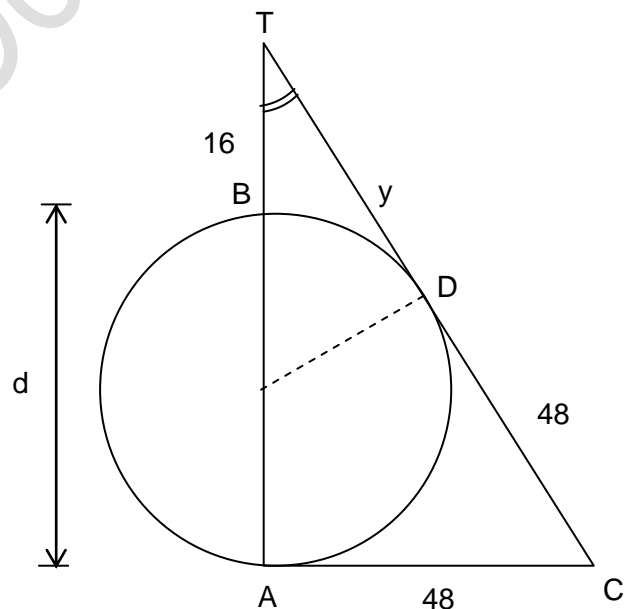
$$dy = 48 \times 32 \dots\dots(2)$$

$$16d^2(16 + d) = d^2y^2 \quad (\text{From (1)})$$

$$16d^2(16 + d) = 48^2 \cdot 32^2$$

$$d^2(16 + d) = 48^2 \times 64$$

Clearly, $d = 48$ satisfies



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26. Min. value occurs for $x = 26, y = 6, z = 4$
 \therefore Answer = 50

BHARAT

27. 1 2 3 4 5 6

Choose two positions for the A's out of 1,2,4,6 in 4C_2 ways.

To arrange the rest of four letters use Principle of Inclusion and Exclusion.

Required number of ways

$$= {}^4C_2 (4! - 3! - 3! + 2!) = 84 \text{ ways}$$

28. $r = \frac{\Delta}{S} = 15$

$$r_1 + r_2 + r_3 = r = 15 \text{ (Property)}$$

29. $a = 2c$

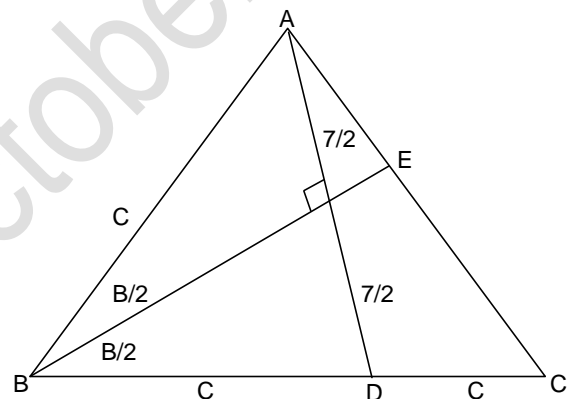
$$BE = \frac{2ac}{a+c} \cos\left(\frac{B}{2}\right) \text{ (Property)}$$

$$\Rightarrow \cos\frac{B}{2} = \frac{27}{4c} \Rightarrow c \cos\frac{B}{2} = \frac{27}{4} = BP$$

$$\Rightarrow \text{Area}(\Delta ABC) = 4 \text{ Ar}(\Delta ABP)$$

$$= 4 \times \frac{1}{2} \times \frac{27}{4} \times \frac{7}{2} = \frac{63}{4} = 47.25$$

Answer 47



30. Total number of values of $n = 64$.



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