FIITJEE

Solutions to JEE(Main) -2024

Test Date: 4^{th} April 2024 (First Shift)

MATHEMATICS, PHYSICS & CHEMISTRY

Paper – 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (MPC) has 30 questions. The maximum marks are 300.
3. This question paper contains three parts**. Part-A** is Mathematics, **Part-B** is Physics and **Part-C** is Chemistry. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20, 31 – 50, 61 – 80)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (21 – 30, 51 – 60, 81 – 90)** contains 30 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

Note: For the benefit of the students, specially the aspiring ones, the question of JEE(Main), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test.

PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q1. Let $f(x)=\begin{cases}-2, & -2\le x\le 0\\ x-2, & 0<x\le 2\end{cases}$ and h(x) = f(|x|) + |f(x)|. Then $\int_{-2}^{2} h(x)\,dx$ is equal to

(A) 4 (B) 6
(C) 1 (D) 2

Ans. D

Sol. Here, $f(x)=\begin{cases}-2 & , -2\le x\le 0\\ x-2 & , 0<x\le 2\end{cases}$

$h(x)=f(|x|)+|f(x)|$

$f(|x|)=\begin{cases}-x-2 & , -2\le x<0\\ x-2 & , 0<x\le 2\end{cases}$

$|f(x)|=\begin{cases}2 & ; -2\le x<0\\ -(x-2); & 0\le x\le 2\end{cases}$

So, $h(x)\begin{cases}(-x-2)+2 & ; -2\le x<0\\ x-2-x+2 & ; 0\le x\le 2\end{cases}$

$\int_{-2}^{2} h(x)dx=\int_{-2}^{0}(-x)dx+\int_{0}^{2}0.dx$

$=-\frac{x^2}{2}\Big|_{-2}^{0}+0$

$=2$

Q2. Three urns A, B and C contain7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(A) $\frac{5}{18}$ (B) $\frac{5}{16}$
(C) $\frac{4}{17}$ (D) $\frac{7}{18}$

Ans. A

Sol. Probability that ball drawn is black

$=\frac{1}{3}\times\frac{5}{12}+\frac{1}{3}\times\frac{7}{12}+\frac{1}{3}\times\frac{6}{12}$

Probability that ball drawn from bag A

is $=\dfrac{\frac{1}{3}\times\frac{5}{12}}{\frac{1}{3}\times\frac{5}{12}+\frac{1}{3}\times\frac{7}{12}+\frac{1}{3}\times\frac{6}{12}}$

$=\frac{5}{5+7+6}=\frac{5}{18}$

Q3. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(A) $x^2 + 10x + 16 = 0$
(B) $2x^2 + 11x + 12 = 0$
(C) $4x^2 + 14x + 12 = 0$
(D) $x^2 + 8x + 12 = 0$

Ans. D

Sol. $ax^2 + bx + 1 = 0$
Two roots : 2,6
Let $\alpha = 2, \beta = 6$

So, $\alpha + \beta = \frac{-b}{a} = 8$

$b = -8a$

$\Rightarrow \alpha\beta = 12 = \frac{1}{a}$

So, $b = -8 \times \frac{1}{12} = -\frac{2}{3}$

Now, for new roots quadratic equation :

$\Rightarrow x^2 - \left(\frac{1}{2a+b} + \frac{1}{6a+b}\right)x + \left(\frac{1}{2a+b}\right)\cdot\left(\frac{1}{6a+b}\right) = 0 \Rightarrow x^2 - (-2-6)x + (-2)(-6) = 0$

So, $x^2 + 8x + 12 = 0$

Q4. The vertices of a triangle are A(–1, 3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

(A) $x + y - (2 - \sqrt{2}) = 0$
(B) $x + y + (2 - \sqrt{2}) = 0$
(C) $-x + y - (2 - \sqrt{2}) = 0$
(D) $x - y - (2 + \sqrt{2}) = 0$

Ans. A

Sol. Uliven, A (-1, 3)
B (-2, 2)
C (3, -1)

So, side AB ≡ y = x + 4
side CA ≡ y = -x + 2
side BC ≡ 5y = -3x + 4

origin lies outside of the triangle ABC

Now, Distance of AB from O(0,0) is $\frac{4}{\sqrt{2}} = 2\sqrt{2}$

Distance of BC from O(0,0) is $\frac{4}{\sqrt{34}}$

Distance of CA from O(0,0) is $\frac{2}{\sqrt{2}} = \sqrt{2}$

Each side of ΔABC shifted by one unit inwards distance of new side origin are

$2\sqrt{2} - 1, \frac{4}{\sqrt{34}} + 1$ and $\left(\sqrt{2} - 1\right)$ respectively

Clearly new position of CA is nearest to the O(0,0)
So, equation of CA new position is

$x + y - 2 + \sqrt{2} = 0$

or, $x + y - \left(2 - \sqrt{2}\right) = 0$

Q5. Let f(x) = $x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function g(x) such that (gof) (x) = x for all $x \in R$. Then the value of 8g'(2) is

(A) 16 (B) 4
(C) 8 (D) 2

Ans. A

Sol. $(g\ of)(x) = g(f(x)) = x$

So, $g'(f(x)) \cdot f'(x) = 1$

$g'(f(x)) = \frac{1}{f'(x)}$

For g' (2)
put f (x) = 2
So, $x^5 + 2e^{x/4} = 2$
So, x = 0

Now, $g'(2) = \frac{1}{f'(0)} = 2$

Now, 8g' (2) = 16

Q6. If the system of equations

$x + (\sqrt{2}\sin\alpha)y + (\sqrt{2}\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to

(A) $\frac{7\pi}{24}$ (B) $\frac{3\pi}{4}$
(C) $\frac{5\pi}{24}$ (D) $\frac{11\pi}{24}$

Ans. C

Sol. For Non-trivial solution we know,

$$\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

Where $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow -1 + \sqrt{2}\sin 2\alpha - \sqrt{2}\cos 2\alpha = 0$

$\sin\left(2\alpha - \frac{\pi}{4}\right) = \frac{1}{2}$

$2\alpha - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{6}$; $n \in I$

Given $\alpha \in \left(0, \frac{\pi}{2}\right)$

So, $\alpha = \frac{5\pi}{24}$

Q7. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to

(A) 633 (B) 3133
(C) 931 (D) 6131

Ans. B

Sol. $\left(2^{1/5}+5^{1/3}\right)^{15}$

$T_{r+1}=15c_r 2^{\frac{15-r}{5}}\times 5^{\frac{r}{3}}$

For rational numbers $\dfrac{15-r}{5}$ and $\dfrac{r}{3}$ should be integers

So, r = 0,5,10,15 if $\dfrac{15-r}{5}$ is integers

r = 0,3,6,9,12,15 If $\dfrac{r}{3}$ is Integers

So, common values are 0,15

Now, sum of rational terms =2^3 + 5^5 = 3133

Q8. Let the first three terms 2, p and $q\neq 2$, of a G.P. be respectively the 7th, 8th and 13th terms of an A.P. If the 5th term of the G.P. is the nth terms of the A.P., then n is equal to

(A) 163 (B) 151
(C) 177 (D) 169

Ans. A

Sol. 2, p and q with $q\neq 2$ in G.P

So, $p^2 = 2q$ ……(i)

Let first term = a

Common difference = d

So, $t_7 = a + 6d=2$ …..(ii)

$t_8 = a+7d = p$ …..(iii)

$t_{13} = a + 12d = q$ …..(iv)

From (ii) and (iii)

d = p-2

From (ii) and (iv)

6P = q + 10 ……(v)

From (i) and (v)

We have, p = 2 or p = 10

But if p = 2 then q = 2 :Rejected

When p = 10 we have q = 50

So, G.P is

2, 10, 50, ……

From (ii) and (iii)

D = 8, a = -48

Now, 5th term of G.P = nth term of A.P

$\Rightarrow(2)5^4=-46+(n-1)8$

So, n = 163

Q9. Let the sum of the maximum and the minimum values of the function f(x) = $\dfrac{2x^2-3x+8}{2x^2+3x+8}$ be $\dfrac{m}{n}$, where gcd (m, n) = 1. Then m + n is equal to

(A) 195 (B) 217
(C) 201 (D) 182

Ans. C

Sol. Given, $f(x)=\dfrac{2x^2-3x+8}{2x^2+3x+8}\Rightarrow$; $x\in R$

$2(y-1)x^2+3(y+1)x+8(y-1)=0$ For $\forall\ x\in R$

So, $D\geq 0$

$9(y+1)^2 - 64(y-1)^2 \geq 0$

$\Rightarrow y \in \left[\frac{5}{11}, \frac{11}{5}\right]$

Now, sum of maximum and minimum value of f (x) is $\frac{146}{55} = \frac{m}{n}$

So, m + n = 201

Q10. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points P_6, P_7,…,P_{11} on the side BC and 7 points P_{12}, P_{13}, …, P_{18} on the side CA of the triangle. The number of triangles, that can be formed using the points P_1, P_2,…..,P_{18} as vertices, is

(A) 771 (B) 776
(C) 796 (D) 751

Ans. D

Sol. Number of triangles $= 18_{C_3} - \left(5_{C_3} + 6_{C_3} + 7_{C_3}\right)$

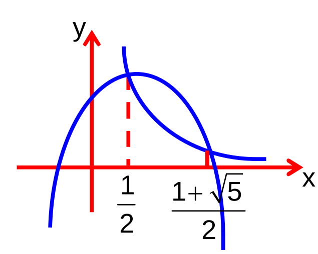
= 816 - 65
= 751

Q11. One of the points of intersection of the curves y = 1 + 3x – 2x^2 and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}(l\sqrt{5} + m) - n\log_e(1+\sqrt{5})$, where l, m, $n \in N$. Then l + m + n is equal to

(A) 32 (B) 31
(C) 30 (D) 29

Ans. C

Sol. Points of Intersection of curves



$y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is

$1 + 3x - 2x^2 = \frac{1}{x}$

$\Rightarrow x = \frac{1}{2},\ x = \frac{1 \pm \sqrt{5}}{2}$

Now, from graph,

So, Area $= \int_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}} \left(1 + 3x - 2x^2 - \frac{1}{x}\right) dx$

$= \left[x + \frac{3x^2}{2} - \frac{2x^3}{3} - \ell nx\right]_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}$

$= \frac{1}{24}\left[14\sqrt{5} + 15\right] - \ell n\left(\sqrt{5} + 1\right)$

So, $\ell = 14$, $m = 15$, $n = 1$

So, $\ell + m + n = 30$

Q12. A square is inscribed in the circle x^2 + y^2 – 10x – 6y + 30 = 0. One side of this square is parallel to y = x + 3. If (x_i, y_i) are the vertices of the square, then $\sum(x_i^2 + y_i^2)$ is equal to

(A) 156 (B) 152
(C) 160 (D) 148

Ans. B

Sol. We know that,
Diagonal of square makes 45° angle with side let slope of diagonal = m

So, $\tan 45^\circ = \left|\frac{m-1}{1+m}\right|$

So, m = not defined and m = 0
So, diagonals : x = 5 and y = 3
This is passing through centre
Now, for x = 5

$x^2 + y^2 - 10x - 6y + 30 = 0$ is

$y^2 - 6y + 5 = 0 \Rightarrow y = 1,5$

So, Two vertices of square is (5,1) and (5,5)
Now, for y = 3

$x^2 + y^2 - 10x - 6y + 30 = 0$ is

$x^2 - 10x + 21 = 0 \Rightarrow x = 3,7$

So, other two vertices of square is (3,3) and (7,3)

So, $\sum\left(x_i^2 + y_i^2\right) = 25 + 1 + 25 + 25 + 9 + 9 + 49 + 9 = 152$

Q13. Let $\alpha \in (0, \infty)$ and A = $\begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. If dat(adj(2A – A^T)) = 2^8, then (dat(A))^2 is equal to

(A) 16 (B) 36
(C) 49 (D) 1

Ans. A

Sol. $\left|\text{adj}\left(2A - A^T\right)\text{adj}\left(A - 2A^T\right)\right| = \left|\text{adj}\left\{\left(A - 2A^T\right)\left(2A - A^T\right)\right\}\right|$

$= \left|\text{adj}\left\{-\left(2A - A^T\right)^T \cdot \left(2A - A^T\right)\right\}\right|$

$= \left|\text{adj}\left(-B^T \cdot B\right)\right|$ Where B = 2A - A^T

$= \left|-B^T \cdot B\right|^2 = \left|-B^T\right|^2 \left|B\right|^2 = \left|B\right|^4$

Now, $2^8 = \left|2A - A^T\right|^4$

So, $\left|2A - A^T\right| = \pm 4$

$\Rightarrow \begin{vmatrix} 1 & 3 & 2\alpha \\ 0 & 0 & 1 \\ -\alpha & 1 & 2 \end{vmatrix} = \pm 4$

So, $\alpha = 1$ or $\alpha = -\frac{5}{3}$ (Rejected)

So, $|A| = \alpha - 5 = 4$

So, $|A|^2 = \left(\det(A)\right)^2 = 16$

Q14. If the domain of the function $\sin^{-1}\left(\frac{3x - 22}{2x - 19}\right) + \log_e\left(\frac{3x^2 - 8x + 5}{x^2 - 3x - 10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to

(A) 95 (B) 100
(C) 97 (D) 98

Ans. C

Sol. For $\sin^{-1}\dfrac{3x-22}{2x-19}$ Domain is

$-1 \le \dfrac{3x-22}{2x-19} \le 1 : x \in \left(3, \dfrac{41}{5}\right]$

For $\log_e\left(\dfrac{3x^2-8x+5}{x^2-3x-10}\right)$ Domain is

$\dfrac{3x^2-8x+5}{x^2-3x-10} > 0:$

$x \in (-\infty,-2) \cup \left(1, \dfrac{5}{3}\right) \cup (5,\infty)$

So, Domain of $\sin^{-1}\dfrac{3x-22}{2x-19} + \log_e \dfrac{3x^2-8x+5}{x^2-3x-10}$ is intersection of $x \in \left(3, \dfrac{41}{5}\right]$ and

$x \in (-\infty,-2) \cup \left(1, \dfrac{5}{3}\right) \cup (5,\infty)$

So, $x \in \left(5, \dfrac{41}{5}\right]$

So, $3\alpha + 10\beta = 97$

Q15. If the solution y = y(x) of the differential equation
$(x^4 + 2x^3 + 3x^2 + 2x + 2)\, dy - (2x^2 + 2x + 3)\, dx = 0$
satisfies $y(-1) = -\dfrac{\pi}{4}$, then y(0) is equal to

(A) $-\dfrac{\pi}{12}$ (B) $\dfrac{\pi}{2}$

(C) $\dfrac{\pi}{4}$ (D) 0

Ans. C

Sol. Here, $\left(x^4 + 2x^3 + 3x^2 + 2x + 2\right)dy - \left(2x^2 + 2x + 3\right)dx = 0$

$\Rightarrow \left(x^4 + 2x^3 + 3x^2 + 2\right)dy = \left(2x^2 + 2x + 3\right)dx$

$$\int dy = \int \frac{\left(2x^2+2x+3\right)}{\left(x^4+2x^3+3x^2+2\right)}dx$$

$$= \int \left\{\frac{1}{x^2+1} + \frac{1}{(x+1)^2+1}\right\}dx$$

$y(x) = \tan^{-1}x + \tan^{-1}(x+1) + c$

Now, $y(-1) = -\dfrac{\pi}{4}$

So, $\dfrac{-\pi}{4} = -\dfrac{\pi}{4} + 0 + c$

$\Rightarrow c = 0$

So, $y(x) = \tan^{-1}x + \tan^{-1}(x+1)$

Hence, $y(0) = \dfrac{\pi}{4}$

Q16. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}$ + $\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is

(A) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(B) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

(C) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(D) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

Ans. D

Sol. Let, $\hat{c}=x\hat{i}+y\hat{j}+z\hat{k}$

$|\hat{c}|=1 \Rightarrow x^2+y^2+z^2=1$......(i)

Given, angle between $\hat{c}$ and $2\hat{i}+2\hat{j}-\hat{k}$ is 60°

$\Rightarrow \cos 60^\circ = \frac{2x+2y-z}{3}$

So, $4x+4y-2z=3$ .....(ii)

and angle between $\hat{c}$ and $\left(\hat{i}-\hat{k}\right)$ is 45°

$\Rightarrow \cos 45^\circ = \frac{x-z}{\sqrt{2}}$

So, x – z = 1 ......(iii)

From (ii) and (iii)

$4y+2z+1=0$

$y=-\left(\frac{2z+1}{4}\right)$

Q17. Let the point, on the line passing through P(1, –2, 3) and Q(5, –4, 7), father from the origin and at a distance of 9 units from the point P, be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to

(A) 165
(B) 150
(C) 155
(D) 160

Ans. C

Sol. Equation of line passes through

P (1, -2,3) and Q (5,-4,7) is

$\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z-3}{4}$

point on it which are at unit distance from P are

$(1\pm 6, -2\pm 3, 3\pm 6)$

$\equiv R(7,-5,9)$ or $S(-5,1,-3)$

So, OR = $\sqrt{49+25+81}$

OS = $\sqrt{25+1+9}$

R is farther from origin

Hence, Required point is (7,-5,9)

So, $\alpha^2+\beta^2+y^2=155$

Q18. Let $f: R \to R$ be a function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2}, & x<0\\ \alpha, & x=0\\ \dfrac{\beta\sqrt{1-\cos x}}{x}, & x>0\end{cases}$$

where $\alpha,\beta \in R$. If f is continuous at x = 0, then $\alpha^2+\beta^2$ is equal to

(A) 3 (B) 48
(C) 12 (D) 6

Ans. C

Sol. For f(x) is continuous at x = 0

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = f(0)$$

So, $\alpha = \lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2} = \lim\limits_{x\to 0}\dfrac{\beta\sqrt{1-\cos x}}{x}$

$\alpha = 2 = \dfrac{\beta}{\sqrt{2}} \Rightarrow \alpha = 2,\ \beta = 2\sqrt{2}$

So, $\left|\alpha^2+\beta^2\right| = 12$

Q19. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2+|z|=0$, $z\in C$. Then $4(\alpha^2+\beta^2)$ is equal to

(A) 4 (B) 6
(C) 8 (D) 2

Ans. A

Sol. $\bar{z}^2+|z|=0$ ……… (i)

$z^2+|\bar{z}|=0$ ………. (ii)

From (i) and (ii)

$\Rightarrow \bar{z}^2 = z^2 \qquad \{|z|=|\bar{z}|\}$

So, $z=\bar{z}$ or $z=-\bar{z}$

Now, $I(z)=0$ or $R(\bar{z})=0$

If I(z) = 0 $\Rightarrow z = x,\ x\in R-\{0\}$

From (i) $x^2 + |x| = 0 \Rightarrow x = 0$ only
(Rejected)

If $R(z)=0 \Rightarrow z = iy,\ y\in R-\{0\}$

From (i) $-y^2+|y|=0$

$y=\pm$ as $y\neq 0$

Hence, z = $\pm i$

So, sum : $\alpha = i - i = 0$

Project : $\beta = (i)(-i) = 1 \Rightarrow 4(\alpha^2+\beta^2) = 4$

Q20. Let $\alpha,\beta\in R$. Let the mean and the variance of 6 observation –3, 4, 7, –6, α,β be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is

(A) $\dfrac{13}{3}$ (B) $\dfrac{16}{3}$
(C) $\dfrac{14}{3}$ (D) $\dfrac{11}{3}$

Ans. A

Sol. $\text{Mean} = 2 = \dfrac{-3+4+7+(-6)+\alpha+\beta}{6}$

$12 = 2+\alpha+\beta$

$\alpha+\beta = 10$ .... (i)

Again variance

$23 = \dfrac{9+16+49+36+\alpha^2+\beta^2}{6} - 2^2$

$\alpha^2+\beta^2 = 52$ ......(ii)

from (i) and (ii)

$\alpha = 6, \beta = 4$ or $\alpha = 4, \beta = 6$

Now, Mean deviation about mean.

$= \dfrac{|-3-2|+|4-2|+|7-2|+|-6-2|+|6-2|+|4-2|}{6}$

$= \dfrac{13}{3}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q21. If the shortest distance between the lines $\dfrac{x+2}{2} = \dfrac{y+3}{3} = \dfrac{z-5}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-2}{-3} = \dfrac{z+4}{2}$ is $\dfrac{38}{3\sqrt{5}}$ k, and $\int_0^k [x^2]\, dx = \alpha - \sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to_____.

Ans. 48

Sol. $L_1 : \dfrac{x+2}{2} = \dfrac{y+3}{3} = \dfrac{z-5}{4}$ ......(i)

$L_2 : \dfrac{x-3}{1} = \dfrac{y-2}{-3} = \dfrac{z+4}{2}$ .....(ii)

Shortest distance between L_1 and L_2

is $= \dfrac{\begin{vmatrix} 5 & 5 & -9 \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}}{\sqrt{18^2+0^2+(-9)^2}}$

$= \dfrac{19}{\sqrt{5}}$ So, $k = \dfrac{3}{2}$

Now, $\int_0^k [x^2] dx = \int_0^{\frac{3}{2}} [x^2] dx$

$= \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + \int_{\sqrt{2}}^{\frac{3}{2}} [x^2] dx$

$= 0 + \left(\sqrt{2}-1\right) + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= 2 - \sqrt{2}$

So, $\alpha = 2$

So, $6\alpha^3 = 48$

Q22. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i} + 2\hat{j} - 7\hat{k}, \overrightarrow{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}, d > 0$. Then the square of the length of the largest side of the triangle ABC is _____.

Ans. 54

Sol. $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\hat{i} + 2\hat{j} - 7\hat{k} + a\hat{i} + b\hat{j} + c\hat{k} = 6\hat{i} + d\hat{j} - 2\hat{k}$

$\Rightarrow (a-5)\hat{i} + (b-d+2)\hat{i} + (c-5)\hat{k} = 0$

$\Rightarrow a = 5,\ b - d + 2 = 0, c = 5$

Now, Area of $\Delta ABC = \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right|$

$$\pm 15\sqrt{2} = \frac{1}{2}\begin{vmatrix} i & j & k \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix}$$

$\pm 30\sqrt{2} = \sqrt{(7d-4)^2 + 1600 + (d-12)^2}$

d = 2

So, a = 5, b = 0 and c = 5

So, $|\overrightarrow{AB}| = \sqrt{54}$ (largest side)

$|\overrightarrow{BC}| = \sqrt{50}$ and $|\overrightarrow{AC}| = \sqrt{44}$

So, $|\overrightarrow{AB}|^2 = 54$

Q23. Let $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + \ldots,$

$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + \ldots$

Then $\frac{2b}{a^2}$ is equal to_____.

Ans. 8

Sol. $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + \ldots\ldots..$

$= 1 + \sum_{n=2}^{\infty} \frac{{}^nC_2}{(n+1)!}$

$= 1 + \frac{1}{2}\sum_{n=2}^{\infty} \frac{n(n+1) - 2(n+1) + 2}{(n+1)!}$

$= 1 + \frac{1}{2}\left\{(e-1) - 2(e-2) + 2\left(e - \frac{5}{2}\right)\right\}$

$= \frac{1}{2}e$

and $b = 1 + \frac{1_{C_0} + 1_{C_1}}{1!} + \frac{2_{C_0} + 2_{C_1} + 2_{C_2}}{2!} + $

$= 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + ...$

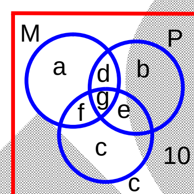
$= e^2$

So, $\frac{2b}{a^2} = 8$

Q24. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to_____.

Ans. 45

Sol.



$125 \le n(M) \le 130$

$85 \le n(P) \le 95$

$75 \le n(C) \le 90$

$n(P \cap C) = 30$

$n(C \cap M) = 50$

$n(M \cap P) = 40$

$\Rightarrow (d + e + f) + 3g = 120$

and a + b + c + d + e + f + g = 210

So, a + b + c = 90 + 2g

Now, $125 \le a + d + f + g \le 130$

$85 \le b + d + e + g \le 95$

$75 \le c + e + f + g \le 90$

$285 \le (a + b + c) + 2(d + e + f) + 3g \le 315$

$285 \le 90 + 2g + 2(120 - 3g) + 3g \le 315$

$285 \le 330 - g \le 315$

$-45 \le -g \le -15$

$15 \le g \le 45$

but $n(P \cap C) = 30$

$\Rightarrow d + g = 30$

$\Rightarrow g \le 30$

Hence, $15 \le g \le 30$

So, M + n = 45

Q25. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1 + \sin x \cos x} dx = \frac{1}{a} \log_e \left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where a, b $\in$ N, then a + b is equal to_____.

Ans. 8

Sol. $I = \int_0^{\pi/4} \frac{\sin^2 x}{1 + \sin x \cos x} dx$

$= \int_0^{\pi/4} \frac{1 - \cos 2x}{2 + \sin 2x} dx$

$$= \int_0^{\pi/4} \frac{1}{2+\sin 2x}dx - \int_0^{\pi/4} \frac{\cos 2x}{2+\sin 2x}dx$$

$$= \int_0^{\pi/4} \frac{\sec^2 x}{2+2\tan^2 x+2\tan x}dx - \frac{1}{2}\int_0^{\pi/4} \frac{2\cos 2x}{2+\sin 2x}dx$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+t+1} - \frac{1}{2}\ell n\left(2+\sin 2x\right)\Big|_0^{\pi/4}$$

$$= \frac{\pi}{6\sqrt{3}} + \frac{1}{2}\ell n\frac{2}{3}$$

So, a = 2 and b = 6
So, a + b = 8

Q26. Let A be a 3 × 3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det(A) is_____.

Ans. 27

Sol. Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

$\Rightarrow$ a + b + c = 3
p + q + r = 3
x + y + z = 3
Such that,
A, b, c, p, q, r, x y, z $\in$ [0,3]
$|A| = (aqz + brx + cpy) - (cqx + bpz + ary)$
For maximum value of | A |,
cqx = 0, bpz = 0 and ary = 0
So that at least one of aqz or brx or cpyis non-zero
Let c = 0, p = 0, and y = 0
$\Rightarrow |A| = aqz + brx = aqz + (3-a)(3-q)(3-z)$
= 27 – 9 (a + q + z) +3 (aq + az + qz) $\le$ 27
for a + q + z = 0
So, maximum value is 27.

Q27. Let A be a square matrix of order 2 such that |A| = 2 and the sum of its diagonal elements is –3. If the points (x, y) satisfying A^2 + xA + y I = O lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is l, then $e^4 + l^4$ is equal to _____.

Ans. **(DROP) Data Insufficient**

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 2$

And $a + d = -3$

Now, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 + x\begin{bmatrix} a & b \\ c & d \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

* we can't determine x and y
Without getting matrix [A]

Q28. Let the solution y = y(x) of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy y(π) = 1. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to_____.

Ans. 7

Sol. $\frac{dy}{dx} - y = 1 + 4\sin x$

I.F = $e^{\int -dx} = e^{-x}$

So, $ye^{-x} = \int e^{-x}(1 + 4\sin x)dx$

$y = -1 - 2(\sin x + \cos x) + ce^{x}$

$y(\pi) = 1$

So, $1 = -1 - 2(0 - 1) + ce^{\pi}$

c = 0

Hence, $y = -1 - 2(\sin x + \cos x)$

$\Rightarrow y\left(\frac{\pi}{2}\right) + 10 = -1 - 2(1 + 0) + 10$

= 7

Q29. If $\lim\limits_{x \to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/3} - (x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$, where gcd(m, n) = 1, then 8m + 12n + equal to _____.

Ans. 100

Sol. $\lim\limits_{x \to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}}$

$\frac{0}{0}$ form

$\lim\limits_{x \to 1} \frac{\{(5x+1) - (x+5)\}\{\sqrt{2x+3} + \sqrt{x+4}\}}{\{(5x+1)^{2/3} + (5x+1)^{1/3}(x+5)^{1/3} + (x+5)^{2/3}\}} \cdot \{(2x+3) - (x+4)\}$

$= 4 \cdot \frac{\sqrt{5} + \sqrt{5}}{(6^{2/3} + 6^{2/3} + 6^{2/3})}$

$= \frac{8.5^{1/3}}{3.(2.3)^{2/3}}$

So, m = 8 and n = 3

Hence, 8m + 12n = 100

Q30. Let the length of the focal chord PQ of the parabola y^2 = 12x be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to_____.

Ans. 72

Sol. Let end points of focal chord are

are $P(3t^2, 6t)$ and $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

Length of focal chord : $3t^2 + 3 + \frac{3}{t^2} + 3 = 15$

$\Rightarrow \left(t+\frac{1}{t}\right)^2 = 5$

So, $t+\frac{1}{t} = \pm\sqrt{5}$

Equation of PQ is $y\left(t-\frac{1}{t}\right) = 2x-6$

Distance from origin $= \left|\frac{6}{\sqrt{4+\left(t-\frac{1}{t}\right)^2}}\right|$

$\Rightarrow P = \left|\frac{6}{\left(t+\frac{1}{t}\right)}\right| = \frac{6}{\sqrt{5}}$ So, $10P^2 = 72$

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q31. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is 25 D. Focal length of each of the convex lens is:

(A) 25 cm (B) 20 cm
(C) 50 cm (D) 500 cm

Ans. B

Sol. Let power of each convex lense is 'P' and focal length is f.
The effective power of combination of 5 identical convex lenses will be

$$P_{effective} = \sum_{i=1}^{5} P_i = 5P$$

$$\Rightarrow 5P = 25D$$

$$P = 5D$$

$$\frac{1}{f} = 5m^{-1}$$

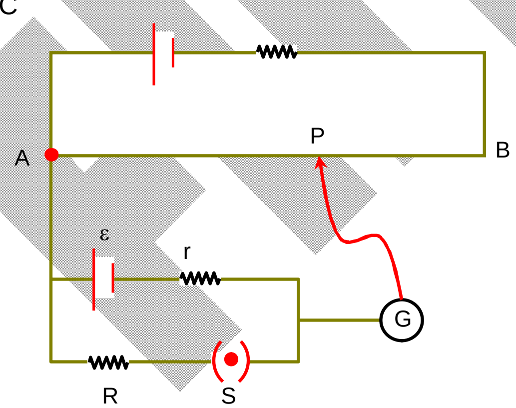
$$f = \frac{1}{5}m = 20\,cm$$

Q32. To measure the internal resistance of a battery, potentiometer is used. For $R = 10\Omega$, the balance point is observed at $\ell = 500$ cm and for $R = 1\Omega$ the balance point is observed at $\ell = 400$ cm. The internal resistance of the battery is approximately:

(A) 0.2Ω (B) 0.4Ω
(C) 0.3Ω (D) 0.1Ω

Ans. C

Sol.



Let potential grade in wire AB is 'x' v/cm

For $R = 10\Omega$, the balance point is 500 cm

$$\frac{\varepsilon}{r+10} \times 10 = x \times 500 \text{ ....(I)}$$

For $R = 1\Omega$, the balance point is 400 cm

$$\frac{\varepsilon}{r+1} \times 1 = x \times 400 \text{ ....(II)}$$

From equation (I) & (II)

$I \div II$

$$\frac{\frac{\varepsilon}{r+10}\times 10}{\frac{\varepsilon}{r+1}\times 1} = \frac{500x}{400x}$$

$r \simeq 0.3\Omega$

Q33. The equation of stationary wave is:

$$y = 2a\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right).$$

Which of the following is NOT correct:

(A) The dimensions of nt is [L]

(B) The dimensions of n is $\left[LT^{-1}\right]$

(C) The dimensions of x is [L]

(D) The dimensions of n/λ is [T]

Ans. D

Sol. $y = 2a\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$

The dimensions of nt is,

$[nt] = [\lambda] = [L]$

The dimensions of n is,

$[nt] = [L]$

$[n] = \left[LT^{-1}\right]$

The dimension of x is,

$[x] = [\lambda] = [L]$

The dimension of n/λ is,

$$\left[\frac{n}{\lambda}\right] = \left[\frac{LT^{-1}}{L}\right] = \left[T^{-1}\right]$$

Q34. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv, respectively. The error in the measurement of the focal length of the convex lens will be:

(A) $\frac{\Delta u}{u} + \frac{\Delta v}{v}$

(B) $f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$

(C) $2f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$

(D) $f^2\left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$

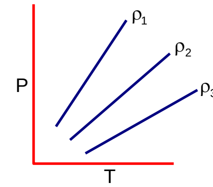
Ans. D

Sol. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Delta f = f^2\left[\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right]$$

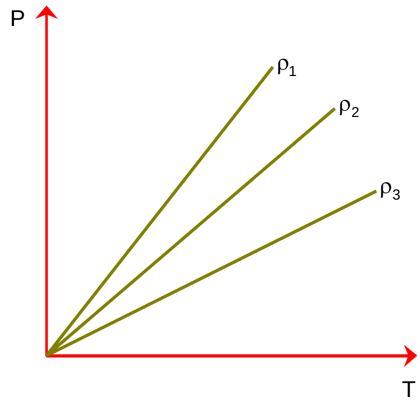
Q35. P-T diagram of an ideal gas having three different densities ρ_1, ρ_2, ρ_3 (in three different cases) is shown in the figure. Which of the following is correct:



(A) $\rho_1 < \rho_2$ (B) $\rho_1 = \rho_2 = \rho_3$
(C) $\rho_1 > \rho_2$ (D) $\rho_2 < \rho_3$

Ans. C

Sol.



$PV = nRT$

$PV = \frac{m}{M}RT$

$P = \frac{n}{v}\frac{RT}{M}$

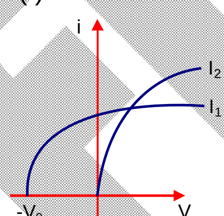
$\left(\rho = \frac{m}{v}\right)$

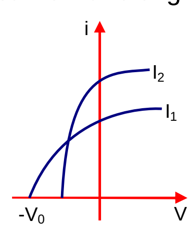
$P = \left(\frac{\rho R}{M}\right)T$

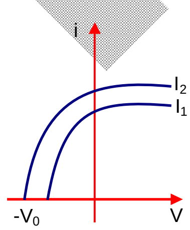
Slope $\propto \rho$

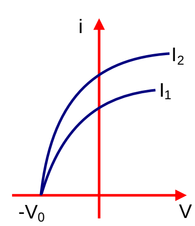
i.e $\rho_1 > \rho_2 > \rho_3$

Q36. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light $(I_1 < I_2)$ of same wavelengths:

(A)


(B)


(C)


(D)


Ans. D

Sol. Stopping potential depends on wavelength of incident light given, wavelength of incident light are same so, stopping potential will be same.
Saturation current is directly proportional to the intensity of incident photons.

Given, Intensity $I_2 > I_1$

So, saturation current corresponding to intensity I_2 will be greater than saturation current corresponding to intensity I_1

Q37. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Then:

(A) the electron path will be circular about the axis.

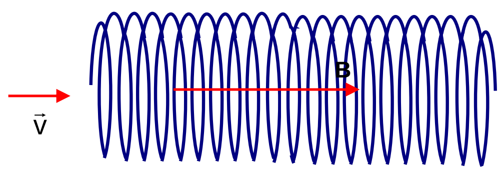
(B) the electron will be accelerated along the axis.

(C) the electron will experience a force at 45^0 to the axis and execute a helical path.

(D) the electron will continue to move with uniform velocity along the axis of the solenoid.

Ans. D

Sol.



Magnetic force

$$\vec{f}_B = q\left(\vec{v} \times \vec{B}\right)$$

Given, $\vec{v}$ is parallel to $\vec{B}$

$$\vec{F}_B = \vec{0}$$

Acceleration = 0

So, the electron will continue to move with uniform velocity along the axis of solenoid.

Q38. In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum. In this case, the source may be connected to:

(a) pure inductor

(b) pure capacitor

(c) pure resistor

(d) combination of an inductor and capacitor

(A) a and b only (B) a, b and d only

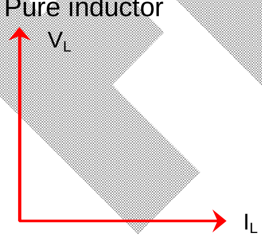
(C) a, b and c only (D) b, c and d only

Ans. B

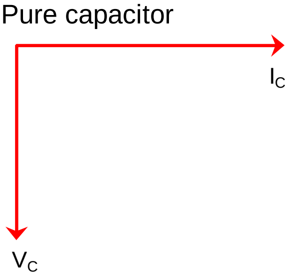
Sol. The instantaneous current is zero, where the instantaneous voltage is maximum.

i.e phase diff b/w current and voltage should be z/2.

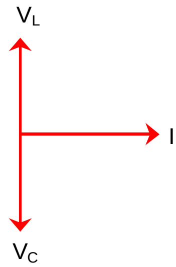
A. Pure inductor



B. Pure capacitor

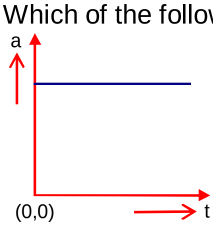


(C) Combination of an inductor and (capacitor)

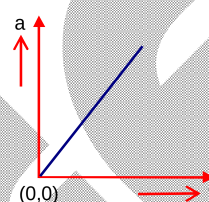


Q39. A wooden block initially at rest on the ground is pushed by a force which increases linearly with time t. Which of the following curve best describes acceleration of the block with time?

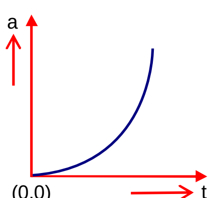
(A)



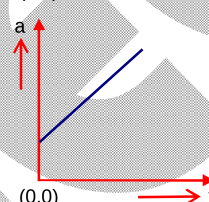
(B)



(C)

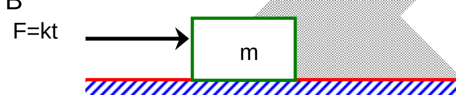


(D)



Ans. B

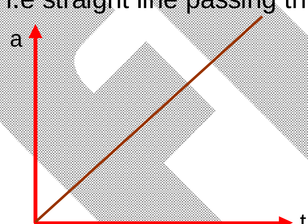
Sol.



$$a = \frac{f}{m} = \frac{kt}{m}$$

$$a = \left(\frac{k}{m}\right)t$$

i.e straight line passing through origin



Q40. The co-ordinates of a particle moving in x-y plane are given by:

$x = 2 + 4t,\ y = 3t + 8t^2$.

The motion of the particle is:

(A) Uniformly accelerated having motion along a parabolic path.
(B) non-uniformly accelerated.
(C) uniformly accelerated having motion along a straight line.
(D) uniform motion along a straight line.

Ans. A

Sol. $x = 2 + 4t$ $\quad\quad$ $y = 3t + 8t^2$

$$v_x = \frac{dx}{dt} = 4 \qquad v_y = \frac{dy}{dt} = 3 + 16t$$

$$a_x = \frac{dv_x}{dt} = 0 \qquad a_y = \frac{dv_y}{dt} = 16$$

$\vec{a} = 16\hat{j}\,m/s^2 \Rightarrow$ uniformly accelerated motion.

Also,

$x = 2 + 4t$ ....(I) & $y = 3t + 8t^2$ ....(II)

From equation (I) & (II)

$$y = 3\left(\frac{x-2}{4}\right) + 8\left(\frac{x-2}{4}\right)^2$$

Above equation is quadratic equation

So, path will be parabolic

The motion of the particle is uniformly accelerated and having motion along a parabolic path.

Q41. A body travels 102.5 m in n^{th} second and 115.0 m in $(n + 2)^{th}$ second. The acceleration is:

(A) 6.25 m/s^2
(B) 12.5 m/s^2
(C) 9 m/s^2
(D) 5 m/s^2

Ans. A

Sol. Displacement for n^{th} second is

$$S_n = u + \frac{a}{2}(2n-1)$$

$$102.5 = u + \frac{a}{2}(2n-1) \text{ ....(I)}$$

$$115.0 = u + \frac{a}{2}\left[2(n+2)-1\right]$$

$$115.0 = u + \frac{a}{2}\left[2n+3\right] \text{ ....(II)}$$

(II) – (I)

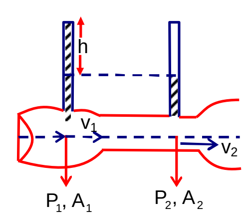
$$12.5 = \frac{a}{2}(4)$$

$$a = 6.25\,m/s^2$$

Q42. Given below are two statements :

Statement I :When speed of liquid is zero everywhere, pressure difference at any two points depends on equation $P_1 - P_2 = \rho g(h_2 - h_1)$

Statement II: In ventury tube shown $2gh = v_1^2 - v_2^2$.



In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both **Statement I** and **Statement II** are incorrect.
(B) **Statement I** is correct but **Statement II** is incorrect.
(C) Both **Statement I** and **Statement II** are correct.
(D) **Statement I** is incorrect but **Statement II** is correct.

Ans. B

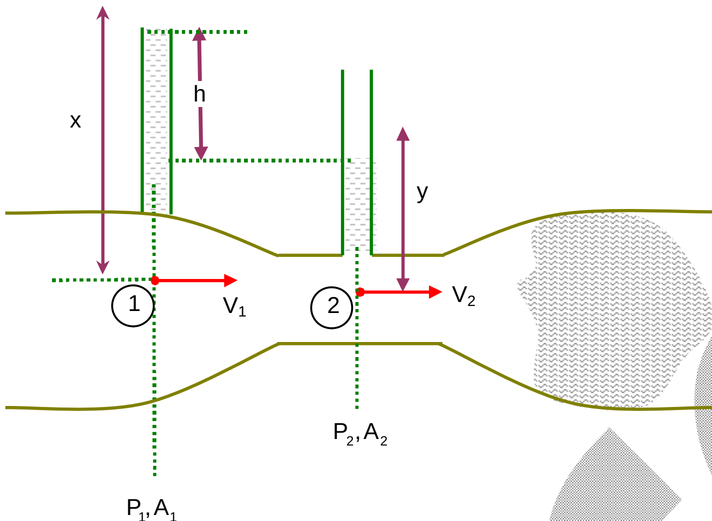
Sol. For statement I

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

If $v_1 \,\&\, v_2 = 0$ every where

$$P_1 - P_2 = \rho g(h_2 - h_1)$$

For statement 2



$P_1 = P_0 + \rho gh$ ………………..(I)

$P_2 = P_0 + \rho gh$ ……………………..(II)

$p_1 - p_2 = \rho g(x - y)$

$P_1 - P_2 = \rho gh$ ………………..(III)

Applying Bernoulli's equation between points 1 & 2

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$(h_1 = h_2) = 0$ points are on reference

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\rho gh = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$v_2^2 - v_1^2 = 2gh$$

Statement '2' is incorrect.

Q43. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are 8Ω and 10Ω respectively. After inserting in a hot bath of temperature 400^0C, the resistance of platinum wire is:

(A) 16Ω (B) 2Ω

(C) 8Ω (D) 10Ω

Ans. A

Sol. $R = R_0(1 + \alpha\Delta T)$

$$R_{100} = R_0\left[1 + \alpha(100 - 0)\right]$$

$$10 = 8\left[1 + \alpha(100)\right]$$

$$\alpha = \frac{1}{400}$$

$$R_{400} = R_0\left[1 + \alpha(400 - 0)\right]$$

$$R_{400} = 8\left[1 + \frac{1}{400} \times 400\right]$$

$$R_{400} = 16\Omega$$

Q44. Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}_0^1n\right)$ and uranium isotope $\left({}_{92}^{235}U\right)$ is correct:

(A) ${}_{56}^{144}Ba + {}_{36}^{89}Kr + 4{}_0^1n$
(B) ${}_{56}^{144}Ba + {}_{36}^{89}Kr + 3{}_0^1n$
(C) ${}_{51}^{153}Sb + {}_{41}^{99}Nb + 3{}_0^1n$
(D) ${}_{56}^{140}Xe + {}_{38}^{94}Sr + 3{}_0^1n$

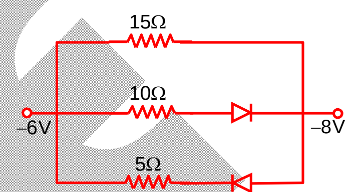
Ans. B

Sol. For a nuclear reaction sum of mass number and sum of atomic number should be balanced.

$${}_{92}^{235}U + {}_0^1n \longrightarrow {}_{56}^{144}Ba + {}_{36}^{89}Kr + 3{}_0^1n$$

$${}_{92}^{235}U + {}_0^1n \longrightarrow {}_{56}^{140}Xe + {}_{38}^{94}Sr + 2{}_0^1n$$

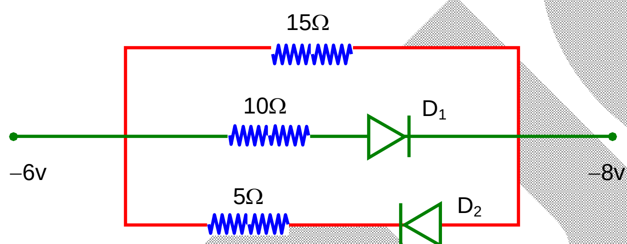
Q45. The value of net resistance of the network as shown in the given figure is:



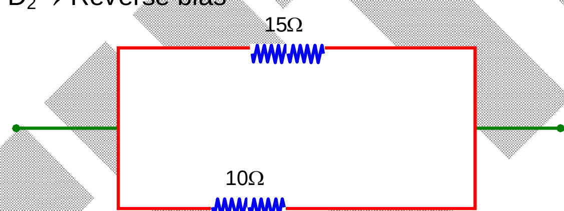
(A) $(15/4)\Omega$
(B) 6Ω
(C) $(5/2)\Omega$
(D) $(30/11)\Omega$

Ans. B

Sol.



$D_1 \rightarrow$ forward bias
$D_2 \rightarrow$ Reverse bias



$$R_{eq} = \frac{15 \times 10}{15 + 10}\Omega = 6\Omega$$

Q46. On celcius scale the temperature of body increases by 40^0C. The increase in temperature on Fahrenheit scale is:

(A) 75^0F
(B) 72^0F
(C) 68^0F
(D) 70^0F

Ans. B

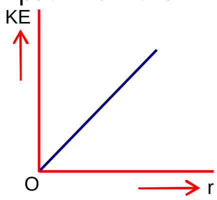
Sol. $$\frac{C-0}{100-0} = \frac{F-32}{212-32}$$

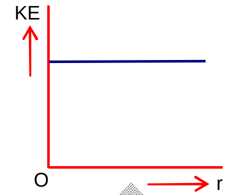
$$F = \frac{9}{5}C + 32$$

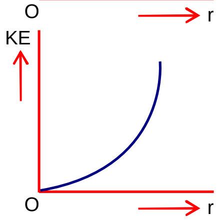
$$\Delta F = \frac{9}{5}(\Delta C)$$

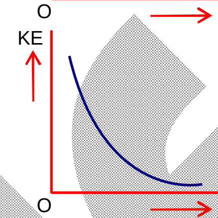
$$\Delta F = \frac{9}{5} \times 40 = 72^0F$$

Q47. An infinitely long positively charged straight thread has a linear charge density λCm^{-1}. An electron revolves along a circular path having axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of electron as a function of radius of circular path from the wire is:

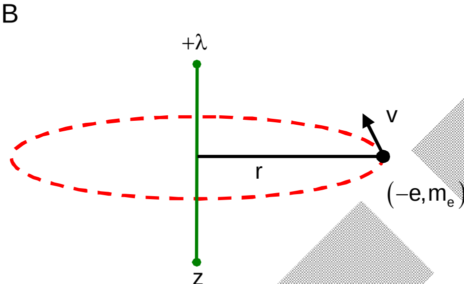
(A)


(B)


(C)


(D)


Ans. B

Sol.


$$F_e = \frac{m_e v^2}{r}$$

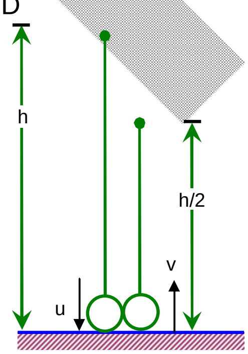
$$e\left(\frac{2K\lambda}{r}\right) = \frac{m_e v^2}{r}$$

$$KE = \frac{1}{2}m_e v^2 = K\lambda e = \text{constant kinetic energy is independent of r.}$$

Q48. If a rubber ball falls from a height h and rebounds upto the height of h/2. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are:

(A) 40%, $\sqrt{2gh}$
(B) 50%, $\sqrt{gh}$
(C) 50%, $\sqrt{\frac{gh}{2}}$
(D) 50%, $\sqrt{2gh}$

Ans. D

Sol.


$\rightarrow$ Velocity of ball before striking the ground

$u = \sqrt{2gh}$

$\rightarrow$ Velocity of ball after striking the ground is

$$v = \sqrt{2g\frac{h}{2}}$$

The percentage loss of total energy of the initial system

$$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} \times 100$$

$$\left(1 - \frac{v^2}{u^2}\right) \times 100$$

$$= \left(1 - \frac{2gh/2}{2gh}\right) \times 100$$

$$= 50\%$$

Q49. The electric field in an electromagnetic wave is given by $\vec{E} = \hat{i}\,40\cos\omega\left(t - \frac{z}{c}\right)NC^{-1}$. The magnetic field induction of this wave is (in SI unit) :

(A) $\vec{B} = \hat{k}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(B) $\vec{B} = \hat{i}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(C) $\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(D) $\vec{B} = \hat{j}\,40\cos\omega\left(t - \frac{z}{c}\right)$

Ans. C

Sol. $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)Nc^{-1}$

i.e electric field is along +x direction and velocity of wave is along +z direction

$$\vec{B} = \frac{\vec{v} \times \vec{E}}{|\vec{v}|^2}$$

$$\vec{B} = \frac{\left(c\hat{k}\right) \times \left(\hat{i}40\cos\omega(t - z/c)\right)}{c^2}$$

$$\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$$

Q50. A metal wire of uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is:

(A) $\frac{2GmM\pi}{L^2}$

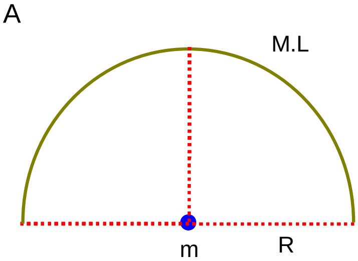
(B) 0

(C) $\frac{GmM\pi^2}{L^2}$

(D) $\frac{GMm\pi}{2L^2}$

Ans. A

Sol.



$\ell = \pi R$

$R = \dfrac{\ell}{\pi}$

Gravitation field intensity at centre of semi ring $(I) = \dfrac{2G\lambda}{R}$

Where λ is linear mass density $\lambda = \dfrac{M}{L}$

Force on mass m due to semi ring

$F = mI$

$F = m\dfrac{2G\frac{M}{L}}{R}$

$F = \dfrac{2Gm\frac{M}{L}}{L/\lambda}$

$F = \dfrac{2GmM\pi}{L^2}$

SECTION - B

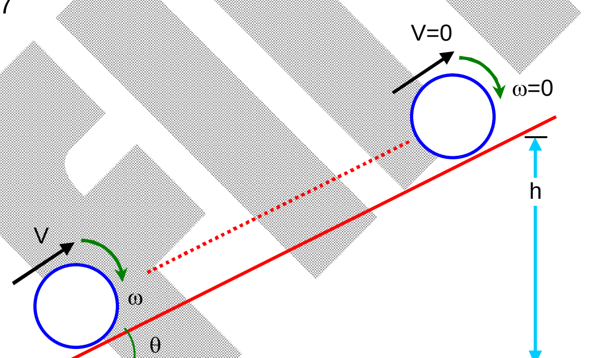
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q51. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches upto maximum heights h_1 and h_2 respectively, above the initial level. The ratio $h_1 : h_2$ is $\dfrac{n}{10}$. The value of n is……….

Ans. 7

Sol.



From concentration of energy theorem

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}mv^2 + \frac{1}{2}mk^2\left(\frac{v^2}{R^2}\right) = mgh$

$h \propto \left(1 + \frac{k^2}{R^2}\right)$

For solid sphere $\frac{k^2}{R^2} = \frac{2}{5}$

For hollow cylinder $\frac{k^2}{R^2} = 1$

$\frac{h_{\text{solid sphere}}}{h_{\text{hollow cylinder}}} = \frac{1+2/5}{1+1} = \frac{7}{10}$

$n = 7$

Q52. An elastic spring under tension of 3 N has a length a. Its length is b under tension 2 N. For its length $(3a-2b)$, the value of tension will be………. N.

Ans. 5

Sol. Let natural length of spring is R

$F = k(\Delta x)$

$3 = k(a-\ell)$ ………………..(I)

$2 = k(b-\ell)$…………………..(II)

$F = k\left[(3a-2b)-\ell\right]$

$F = k\left[3(a-\ell)-2(b-\ell)\right]$

Form equation 1 & 2

$F = k\left[3.\frac{3}{k}-2.\frac{2}{k}\right]$

$F = 5N$

Q53. A soap bubble is blown to a diameter of 7 cm, 36960 erg of work is done in blowing it further. If surface tension of soap solution is 40 dyne/cm then the new radius is…….. cm. Take $\left(\pi = \frac{22}{7}\right)$.

Ans. 7

Sol. Work = change in surface energy

$W = T(\Delta A)$

$W = T\left[8\pi\left(R^2-r^2\right)\right]$

Where T is surface tension
R is final radius

R is initial radius $= \frac{7}{2}$cm

$36960 = 40\times 8\pi\left[R^2-\left(\frac{7}{2}\right)^2\right]$

$R = 7cm$

Q54. Two forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is……..

Ans. 6

Sol. let $|\vec{F}_1| = F$

$|\vec{F}_2| = 3F$

And magnitude of resultant of two forces is equal to the force of larger magnitude.

$|\vec{F}_1+\vec{F}_2| = 3F$

$F^2 + (3F)^2 + 2.F.3F\cos\theta = (3F)^2$

$\cos\theta = -\frac{1}{6}$

$\theta = \cos^{-1}\left(\frac{1}{-6}\right)$

$n = -6$

$|n| = 6$

Q55. A alternating current at any instant is given by $i = \left[6 + \sqrt{56}\sin(100\pi t + \pi/3)\right]A$. The rms value of the current is……….A.

Ans. 8

Sol. $i = 6 + \sqrt{56}\sin\left(100\pi t + \frac{\pi}{3}\right)$

$i_{rms}^2 = \frac{\int i^2 dt}{\int dt} = < i^2 >$

$i_{rms}^2 = < 6^2 > + < 56\sin^2\left(100\pi t + \frac{\pi}{3}\right) > + 2\times 6 \times \sqrt{56}\sin\left(100\pi t + \frac{\pi}{3}\right)$

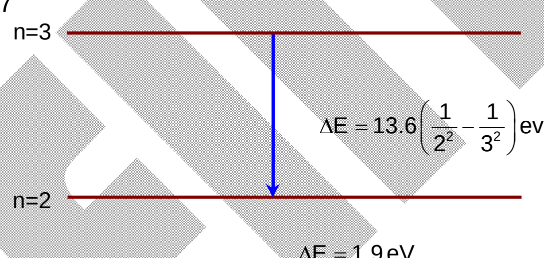
$i_{rms}^2 = 36 + \frac{56}{2} = 64$

$i_{rms} = \sqrt{64} = 8A$

Q56. A hydrogen atom changes its state from n = 3 to n = 2. Due to recoil, the percentage change in the wave length of emitted light is approximately 1×10^{-n}. The value of n is…….
[Given Rhc $= 13.6$ eV, hc $= 1242$ eV nm, $h = 6.6\times 10^{-34}$ Js mass of the hydrogen atom $= 1.6\times 10^{-27}$ kgℓ

Ans. 7

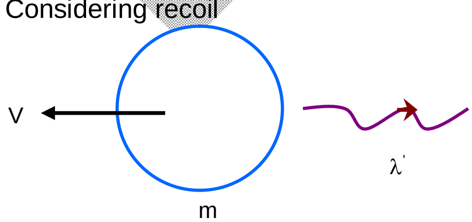
Sol.



Without considering recoil

$\lambda = \frac{hc}{\Delta E}$

Considering recoil



From concentration of momentum

$mv = \frac{h}{\lambda'}$

$$v = \frac{h}{m\lambda'}$$

Also, $\Delta E = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2 + \frac{hc}{\lambda'}$

$$\Delta E(\lambda')^2 - hc(\lambda') - \frac{h^2}{2m} = 0$$

$$\lambda' = \frac{hc \pm \sqrt{h^2c^2 + \frac{4\Delta E h^2}{2m}}}{2\Delta E}$$

λ' can't be negative

$$\lambda' = \frac{hc + hc\sqrt{1 + \frac{2\Delta E}{mc^2}}}{2\Delta E}$$

$$\frac{\lambda'}{(hc/\Delta E)} = \frac{1 + \left(1 + \frac{2\Delta E}{mc^2}\right)^{1/2}}{2}$$

$$\frac{\lambda'}{\lambda} = \frac{1 + \left(1 + \frac{\Delta E}{mc^2}\right)}{2} \qquad \left(\because \lambda = \frac{\Delta E}{mc}\right)$$

$$\frac{\lambda'}{\lambda} = 1 + \frac{\Delta E}{2mc^2}$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta E}{2mc^2}$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{1.9 \times 1.6 \times 10^{-19}}{2 \times 1.67 \times 10^{-27} \times 9 \times 10^{16}}$$

$$\frac{\lambda' - \lambda}{\lambda} = 10^{-9}$$

%ge change

$$\frac{\lambda' - \lambda}{\lambda} \times 100 = 10^{-7}$$

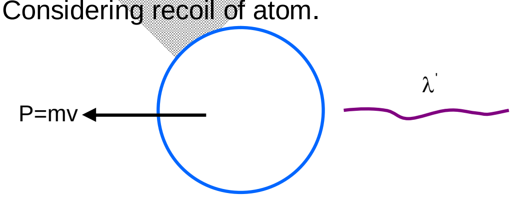
Alternate

$$\Delta E = 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Delta E = 1.9\text{eV}$$

& $\lambda = \frac{hc}{\Delta E}$ ----1

Considering recoil of atom.



$$mv = \frac{h}{\lambda'} \qquad \lambda' = \lambda + \cancel{\Delta}^{\circ}\lambda$$

$$mv = \frac{h}{\lambda}$$

$$v = \frac{h}{m\lambda} = \frac{h}{m\left(\frac{hc}{\Delta E}\right)} = \frac{\Delta E}{mc}$$

$$\Delta E = \frac{1}{2}mv^2 + \frac{hc}{\lambda'}$$

$$\Delta E = \frac{1}{2}m\left(\frac{\Delta E}{mc}\right)^2 + \frac{hc}{\lambda'}$$

$$\Delta E = \frac{\Delta E^2}{2mc^2} + \frac{hc}{\lambda'}$$

$$\frac{hc}{\lambda'} = \Delta E\left(1 - \frac{\Delta E}{2mc^2}\right)$$

$$\frac{hc}{\lambda'} = \frac{hc}{\lambda}\left(1 - \frac{\Delta E}{2mc^2}\right)$$

$$\frac{\lambda'}{\lambda} = \left(1 - \frac{\Delta E}{2mc^2}\right)^{-1} = 1 + \frac{\Delta E}{2mc^2}$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta E}{2mc^2}$$

$$= \frac{1.9 \times 16. \times 10^{-19}}{2 \times 1.67 \times 10^{-27} \times 9 \times 10^{16}}$$

$$\frac{\lambda' - \lambda}{\lambda} = 10^{-9}$$

$$\%ge = 10^{-7}\%$$

Q57. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment. $\lambda_1 = 450\text{ nm}$ and $\lambda_2 = 650\text{ nm}$. The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1 is n. The value of n is………

Ans. 9

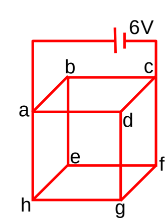
Sol. $n_1\lambda_1 = n_2\lambda_2$

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{450}{650} = \frac{9}{13}$$

The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1.

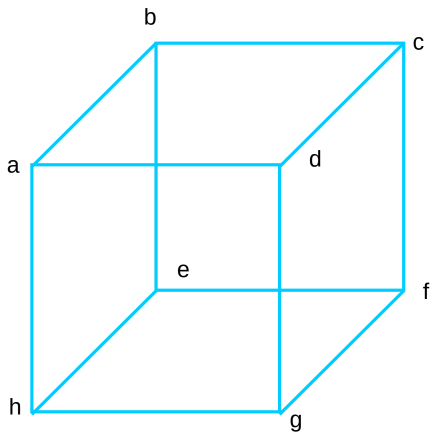
$n_2 = 9$

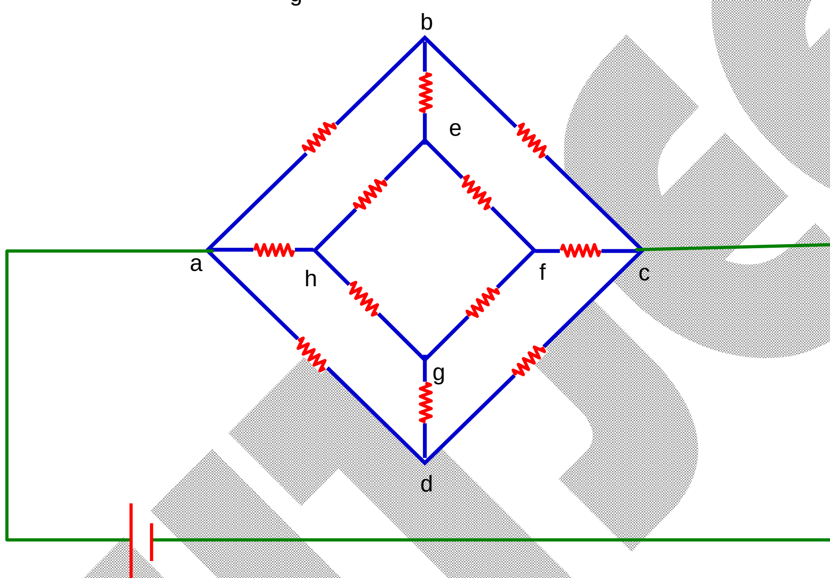
Q58. Twelve wires each having resistance 2Ω are joined to form a cube. A battery of 6V emf is joined across point a and c. The voltage difference between e and f is……….V.



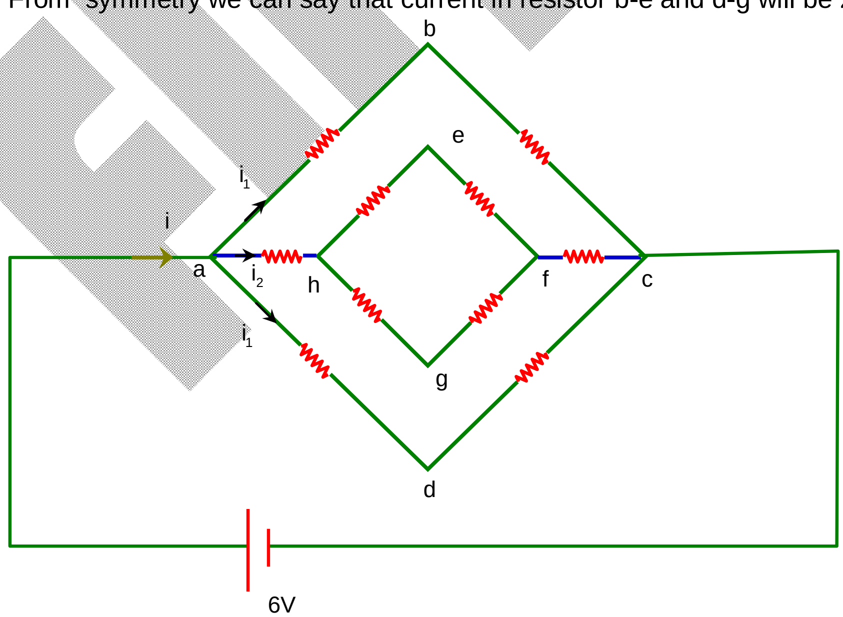
Ans. 1

Sol.





From symmetry we can say that current in resistor b-e and d-g will be zero.



Equivalent resister between a & c is

$= \frac{3R}{4} = \frac{3}{4} \times 2 = 1.5\Omega$

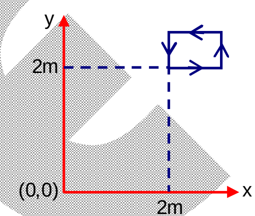
$i = \frac{6}{R_{eq}} = \frac{6}{1.5} = 4A$

$i_2 = \frac{6}{3R} = 1A$

Current through resistance e-f will be $\frac{i_2}{2} = \frac{1}{2}A$

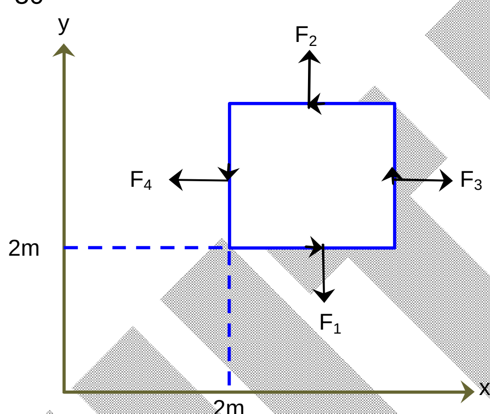
Potential difference $V_{ef} = \frac{1}{2} \times 2 = 1V$

Q59. The magnetic field existing in a region is given by $\vec{B} = 0.2(1+2x)\hat{k}T$. A square loop of edge 50 cm carrying 0.5 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is……mN.



Ans. 50

Sol.



$F_1 = F_2$ (magnetic field is only function of x)

$\rightarrow F_4 = i\ell B = 0.5 \times 0.5 \times [0.2(1+2\times 2)]$

$F_4 = 0.25N$

$F_3 = i\ell B$

$F_3 = 0.5 \times 0.5 \times [0.2(1+2\times 2.5)]$

$F_3 = 0.30N$

Net force $= (F_3 - F_4)N$

$= (0.30 - 0.25)N$

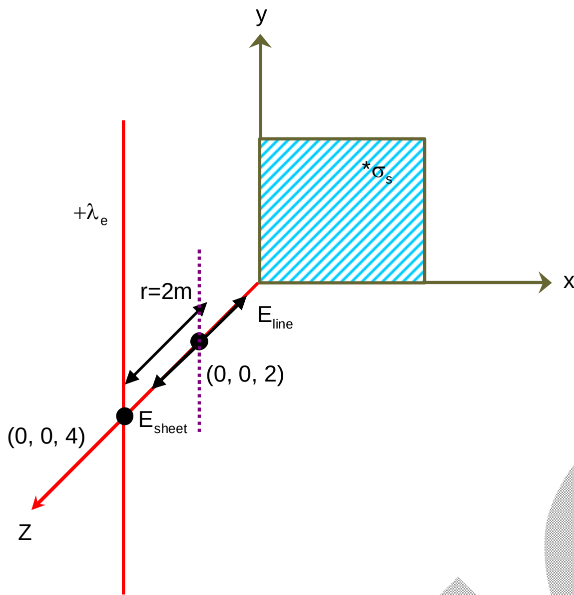
$= 0.05N$

$= 50\,mN$

Q60. An infinite plane sheet of charge having uniform surface charge density $+\sigma_s C/m^2$ is placed on x-y plane. Another infinitely long line charge having uniform linear charge density $+\lambda_e C/m$ is placed at z = 4m plane and parallel to y-axis. If the magnitude values $|\sigma_s| = 2|\lambda_e|$ then at point $(0,0,2)$, the ratio of magnitudes of electric field values due to sheet charge to that of line charge is $\pi\sqrt{n}:1$. The value of n is……..

Ans. 16

Sol.



Ratio of magnitude of electric filed values due to sheet charge to that of line change is

$$\frac{E_{sheet}}{E_{Line}} = \frac{\sigma_s / 2\varepsilon_0}{\lambda_e / 2\pi\varepsilon_0 r}$$

$$= \frac{\sigma_s \pi r}{\lambda_e}$$

$$= \frac{2\lambda_e \pi r}{\lambda_e} \left(\because |\sigma_s| = 2|\lambda_e|\right)$$

$$= 2\pi r$$

$$= 2\pi \times (2)$$

$$= 4\pi = \sqrt{16}\,\pi$$

$$n = 16$$

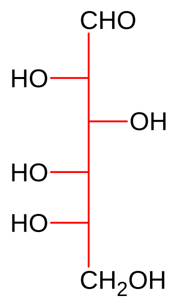
PART – C (CHEMISTRY)

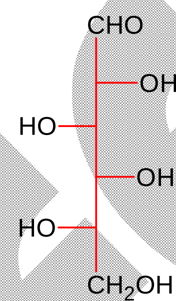
SECTION - A

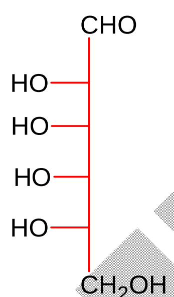
(One Options Correct Type)

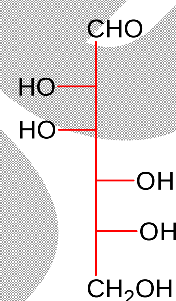
This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

Q61. Which of the following is the correct structure of L-Glucose?

(A) 

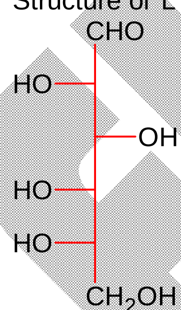
(B) 

(C) 

(D) 

Ans. A

Sol. Structure of L – glucose is



Q62. What will be decreasing order of basic strength of the following conjugate bases?

$^{-}OH.R\overline{O}, CH_3CO\overline{O}, C\overline{l}$

(A) $C\overline{l} > R\overline{O} > ^{-}OH^{-} > CH_3CO\overline{O}$ (B) $R\overline{O} > ^{-}OH > CH_3CO\overline{O} > C\overline{l}$

(C) $^{-}OH > R\overline{O} > CH_3CO\overline{O} > C\overline{l}$ (D) $C\overline{l} > \overline{O}H > R\overline{O} > CH_3CO\overline{O}$

Ans. B

Sol. conjugate base of strong acid are weak bases

Acidic strength

$H-Cl > CH_3COOH > H_2O > R-OH$

Conjugate base strength

$Cl^{-} < CH_3COO^{-} < \overline{O}H < R-O^{-}$

Q63. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is________________.
B, C, N, S, O, F, P, Al, Si
(A) 3 (B) 7
(C) 5 (D) 6

Ans. A

Sol. N,O and F cannot extend their valancies up to their group number due to non- availability of vacant d orbital

Q64. The element which shows only one oxidation state other than its elemental form is:
(A) Nickel (B) Scandium
(C) Titanium (D) Cobalt

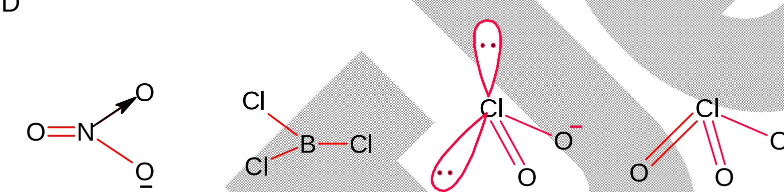
Ans. B

Sol. Sc atomic no 21 $3d^1\ 4s^2$ shows only +3 oxidation state
Ti, Co and Ni show +2, +3 & +4 oxidation state

Q65. Number of molecules / ions from the following in which the central atom is involved in sp^3 hybridzation is____________.
$NO_3^-, BCl_3, ClO_2^-, ClO_3$
(A) 1 (B) 4
(C) 3 (D) 2

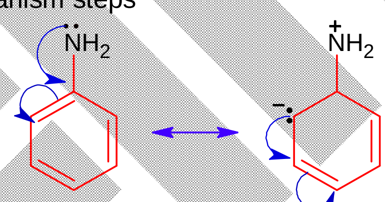
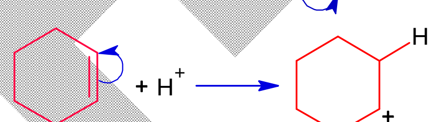
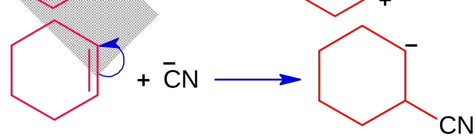
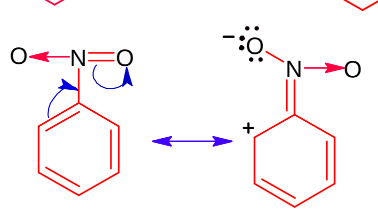
Ans. D

Sol.



Sp^2 Sp^2 Sp^3 Sp^3

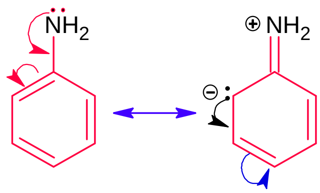
Q66. Match **List I** with **List II:**

List-I Mechanism steps	List-II Effect
(a) 	(I) – E effect
(b) 	(II) – R effect
(c) 	(III) + E effect
(d) 	(IV) + R effect

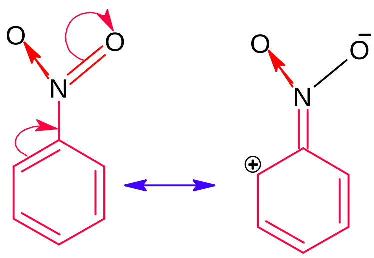
Choose the correct answer from the options given below:
(A) (a) – (II), (b) – (IV), (c) – (III), (d) – (I)
(B) (a) – (III), (b) – (I), (c) – (II), (d) – (IV)
(C) (a) – (IV), (b) – (III), (c) – (I), (d) – (II)
(D) (a) – (I), (b) – (II), (c) – (IV), (d) – (III)

Ans. C

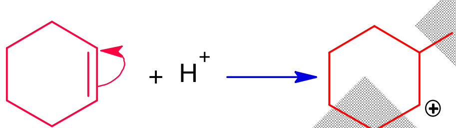
Sol.



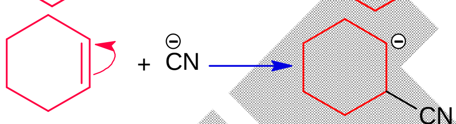
+ R effect NH_2 electron donating



- R effect (NO_2 electron withdrawing)



+ E effect



- E effect

Q67. The molarity (M) of an aqueous solution containing 5.85 g of NaCl in 500 mL water is:
(Given: Molar Mass Na: 23 and Cl: 35.5 $gmol^{-1}$)

(A) 0.2
(B) 2
(C) 4
(D) 20

Ans. A

Sol. Moles of NaCl $\frac{5.85}{58.5} = 0.1$

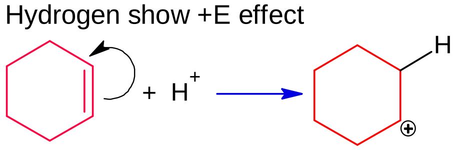
Volume of solution $= \frac{500}{1000} = 0.5\,\text{litre}$

$$\text{Molarity} = \frac{\text{Number of moles}}{\text{V(litre)}} = \frac{0.1}{0.5} = 0.2$$

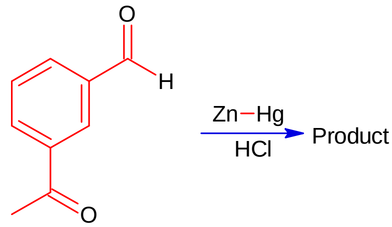
Q68. Which among the following is incorrect statement?
(A) Hydrogen ion (H^+) shows negative electromeric effect
(B) The electromeric effect is, temporary effect
(C) The organic compound shows electromeric effect in the presence of the reagent only.
(D) Electromeric effect dominates over inductive effect

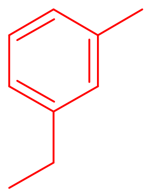
Ans. A

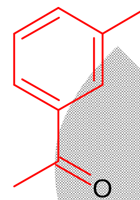
Sol. Hydrogen show +E effect

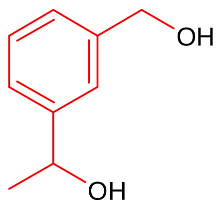


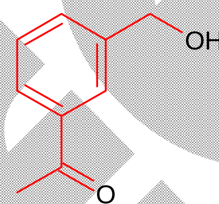
Q69. Identify the product in the following reaction:



(A) 

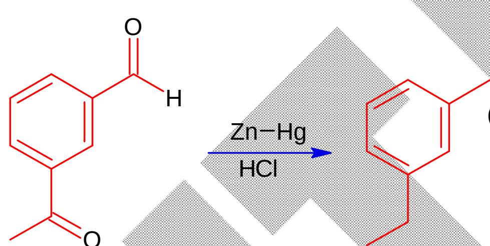
(B) 

(C) 

(D) 

Ans. A

Sol.



(Clemmensen reduction)

Q70. One of the commonly used electrode is calomel electrode. Under which of the following categories,
Calomel electrode comes?
(A) Gas – Ion electrodes
(B) Metal ion – Metal electrodes
(C) Oxidation – Reduction electrodes
(D) Metal – Insoluble Salt – Anion electrodes

Ans. C

Sol. Calomel electrode is metal in soluble salt anion electrode

Q71. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at 25°C?
(A) 10^{-7}
(B) 1
(C) 10^{-14}
(D) 0.5

Ans. D

Sol. $2H^{+}(aq)+2e \longrightarrow H_2(g)$

$$E = E^0 - \frac{0.0591}{n}\log\frac{P_{H_2}}{\left[H^{+}\right]^2}$$

$$0 = 0 - \frac{.0591}{2}\log\frac{P_{H_2}}{\left(10^{-7}\right)^2}$$

$$\log\frac{P_{H_2}}{\left(10^{-7}\right)^2}=0$$

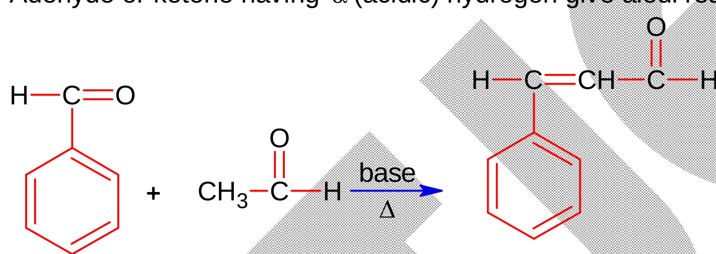
$$\frac{P_{H_2}}{10^{-14}}=1$$

$$P_{H_2}=10^{-14}\text{bar}$$

Q72. Given below are two statements:
Statement I: Acidity of α- hydrogens of aldehydes and ketones is responsible for aldol reaction.
Statement II: Reaction between benzaldehyde and ethanal will NOT give Cross – Aldol product.
In the light of the above statements, choose the most **appropriate** answer from the options given below:
(A) Both **Statement I** and **Statement II** are correct
(B) Both **Statement I** and **Statement II** are incorrect
(C) **Statement I** is incorrect but **Statement II** is correct
(D) **Statement I** is correct but **Statement II** is incorrect

Ans. D

Sol. Adehyde or ketone having α (acidic) hydrogen give alodl reaction



Q73. In the precipitation of the group (III) in qualitative analysis. Ammonium chloride is added before adding ammonium hydroxide to:
(A) Increase concentration of C^{l-} ions
(B) Decrease concentration of –OH ions
(C) Increase concentration of NH_4^+ ions
(D) Prevent interference by phosphate ion

Ans. B

Sol. $NH_4Cl \rightarrow NH_4^+ + Cl^-$

$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

Due to common ion effect of NH_4^+ dissociation of NH_4OH is suppressed so concentration of OH^- decreases so that group III cation can be precipitated.

Q74. Which of the following nitrogen containing compound does not give Lassaigne's test?
(A) Phenyl hydrazine
(B) Glycene
(C) Urea
(D) Hydrazine

Ans. D

Sol. Hydragine $\left(NH_2-NH_2\right)$ have no carbon atom so it does not give lassaigne test.

Q75. Number of complexes from the following with even number of unpaired "d" electrons is__
$[V(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{3+}$, $[Ni(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$
[Given atomic numbers: V= 23, Cr=24, Fe=26, Ni = 28 Cu=29]
(A) 1
(B) 5
(C) 2
(D) 4

Ans. C

Sol. $\left[V\left(H_2O\right)_6\right]^{+3}$ $\qquad$ $V(23)=[Ar]3d^3 4s^2$

$V^{+3} = [Ar]3d^2.sp^3d^2$

Number of unpaired electron = 2 (even)

$[Cr(H_2O)_6]^{+2}$ $\quad Cr(24) \longrightarrow [Ar]3d^5 4s^1$

$Cr^{+2} \longrightarrow [Ar]3d^4.Sp^3d^2$

Number of unpaired electron = 4 (even)

$[Fe(H_2O)_6]^{+3}$ $\quad Fe(26) \longrightarrow [Ar]3d^6 4s^2$

$Fe^{+3} \longrightarrow [Ar]3d^5.sp^3d^2$

Number of unpaired electron 5

$[Ni(H_2O)_6]^{+3}$ $\quad Ni(28) \longrightarrow [Ar]3d^8 4s^2$

$Ni^{+3} \longrightarrow [Ar]3d^7.sp^3d^2$

Number of unpaired electron =3

$[Cu(H_2O)_6]^{+2}$ $\quad Cu(29) \longrightarrow [Ar]3d^{10}\ 4s^1$

$Cu^{+2} \longrightarrow [Ar]\ 3d^9.\ sp^3d^2$

Number of unpaired electron 1

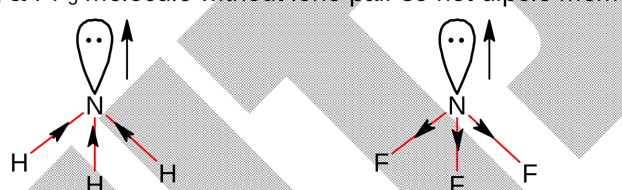
Q76. Which one of the following molecules has maximum dipole moment?

(A) PF_5 (B) CH_4

(C) NF_3 (D) NH_3

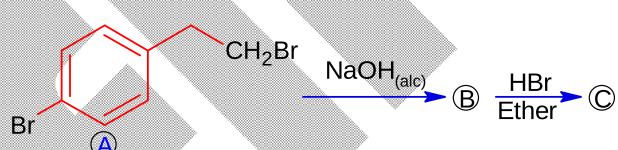
Ans. D

Sol. CH_4 & PF_5 molecule without lone pair so net dipole moment 0



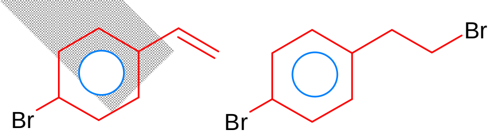
Vector addition of bond moment and lone pair $\qquad$ Vector subtraction of bond moment and lone pair

Q77.



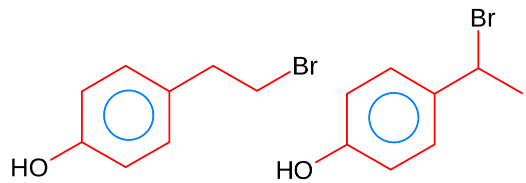
Identify (B) and (C) and how are (A) and (C) related ?

(A)

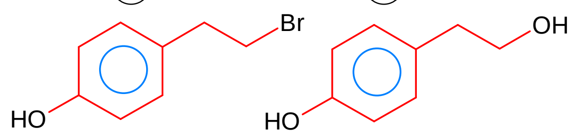


; chain isomers

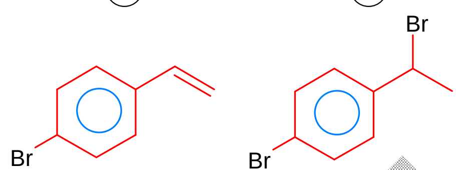
(B) $\qquad$ (C)

(B)  ; Derivative

(B) (C)

(C)  ; Functional group isomers

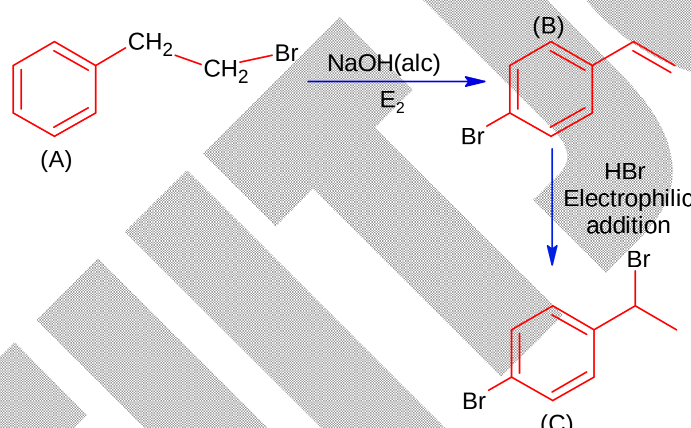
(B) (C)

(D)  ; Position isomers

(B) (C)

Ans. D

Sol.



A & C are position isomer

Q78. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation.

$$CH_3-CH_2-CH_2-Br \xrightarrow{\text{'X'}} \text{Product} \xrightarrow{\text{'Y'}} CH_3-\underset{\underset{Br}{|}}{CH}-CH_3$$

(A) X = conc.alc. NaOH, 80°C, Y= HBr / acetic acid
(B) X = conc.alc. NaOH, 80°C, Y= Br_2 / $CHCl_3$
(C) X = dil.aq. NaOH, 20°C, Y= HBr / acetic acid
(D) X= dil.aq. NaOH, 20°C, Y= Br_2 / $CHCl_3$

Ans. A

Sol.

$$CH_3-CH_2-CH_2-Br \xrightarrow[80\ °C]{x=\text{ Conc. alc NaOH}} CH_3-CH=CH_2$$

$$\downarrow \text{Y= HBr / Acetic acid}$$

$$CH_3-CHBr-CH_3$$

Q79. The correct sequence of ligands in the order of decreasing field strength is :

(A) $^{-}OH > F^{-} > NH_3 > CN^{-}$
(B) $NCS^{-} > EDTA^{4-} > CN^{-} > CO$
(C) $CO > H_2O > F^{-} > S^{2-}$
(D) $S^{2-} > \overline{O}H > EDTA^{4-} > CO$

Ans. C

Sol. According to spectrochemical series correct ligand field strength is

$CO > H_2O > F^{-} > S^{-2}$

Q80. The correct order of first ionization enthalpy values of the following elements is:

(a) O
(b) N
(c) Be
(d) F
(e) B

Choose the correct answer from the options given below:

(A) c < e < a < b < d
(B) e < c < a <b < d
(C) b < d < c < e < a
(D) a < b < d < c<e

Ans. B

Sol. Correct order of 1^{st} ionization energy

B < Be < O < N < F

e < c < a < b < d

SECTION - B

(Numerical Answer Type)

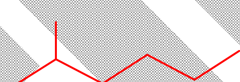
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

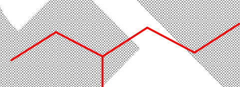
Q81. The number of different chain isomers for C_7H_{16} is__________.

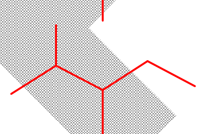
Ans. 9

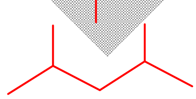
Sol.

(I) 

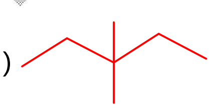
(II) 

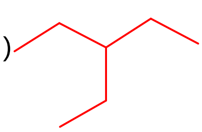
(III) 

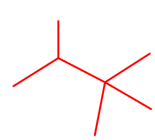
(IV) 

(V) 

(VI) 

(VII) 

(VIII) 

(IX) 

Q82. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.

$$A + B \underset{\text{Step 3}}{\overset{\text{Step 1}}{\rightleftharpoons}} C \xrightarrow{\text{Step 2}} P$$

Some details of the above reaction are listed below.

Step   Rate constant (sec^{-1})   Activation energy ($kJ\ mol^{-1}$)

1	k_1	300
2	k_2	200
3	k_3	Ea_3

If the overall rate constant of the above transformation (k) is given as $k = \frac{k_1 k_2}{k_3}$ and the overall activation energy (Ea) is 400kJ mol^{-1}, then the value of Ea_3 is ____ kJ mol^{-1} (nearest integer)

Ans. 100

Sol. $K = \frac{K_1 K_2}{K_3}$

$$Ae^{\frac{-Ea}{RT}} = \frac{A_1 e^{\frac{-Ea_1}{RT}} A_2 e^{\frac{-Ea_2}{RT}}}{A_3 e^{-Ea_3/RT}}$$

$$Ae^{-\frac{Ea}{RT}} = \frac{A_1 A_2}{A_3} e^{\frac{-(E_{a_1}+E_{a_2}-E_{a_3})}{RT}}$$

$$E_a = E_a + E_{a_2} - E_{a_3}$$

$$400 = 300 + 200 - E_{a_3}$$

$$E_{a_3} = 100$$

Q83. Number of molecules / species from the following having one unpaired electron is______.

$O_2, O_2^{-1}, NO, CN^{-1}, O_2^{2-}$.

Ans. 2

Sol. According to molecular orbital theory

O_2 have 2 unpaired electron

O_2^- have 1 unpaired electron

NO have 1 unpaired electron

CN^- have 0 unpaired electron

O_2^{2-} have 0 unpaired electron

Q84. Consider the following reaction

$MnO_2 + KOH + O_2 \rightarrow A + H_2O$

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment value of B and C_________BM (nearest integer) (Given atomic number of Mn is 25)

Ans. 4

Sol. $MnO_2 + KOH + O_2 \rightarrow K_2MnO_4 + H_2O$

$K_2MnO_4 \xrightarrow{\text{Acidic / Neutral}} KMnO_4 + MnO_2$

$KMnO_4 \rightarrow Mn^{+7} \mu = 0$

$MnO_2 \rightarrow Mn^{+4} \longrightarrow [Ar]3d^3$

Number of unpaired electron=3

$\mu = \sqrt{3(3+2)} = 3.87\,BM$

Nearest integer is (4)

Q85. X g of ethylamine is subjected to reaction with $NaNO_2$ / HCl followed by water, evolvleld dinitrogen gas which occupied 2.24 L volume at STP. X is __________ $\times 10^{-1}$ g.

Ans. 45

Sol. $CH_3CH_2NH_2 \xrightarrow[H_2O]{NaNO_2 / HCl} CH_3 - CH_2 - OH + N_2$

$V_{N_2} = 2.24\,\text{Litre}$

$n_{N_2} = \frac{2.24}{22.4} = 0.1\,\text{mole}$

Moles of $CH_3 - CH_2 - NH_2 = 0.1$

Mass of $CH_3 - CH_2 - NH_3 = 45 \times 0.1$

$= 4.5\,\text{gm}$

$= 45 \times 10^{-1}\,\text{gm}$

Q86. The de-Broglie's wavelength of an electron in the 4^{th} orbit is__________ πa_0
(a_0 = Bohr's radius)

Ans. 8

Sol. Radius of 4^{th} orbit = 16 a_0

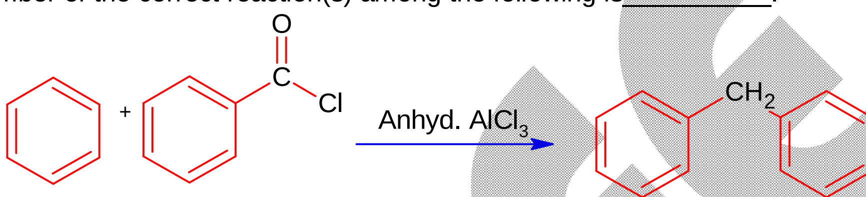
$$2\pi r_n = n\lambda$$

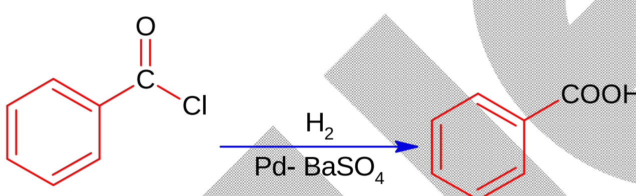
$$2\pi(16a_0) = 4\lambda$$

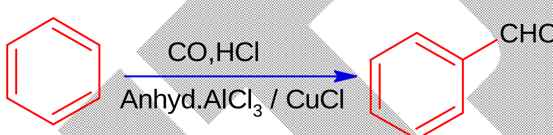
$$\lambda = \frac{2\lambda \times 16a_0}{4}$$

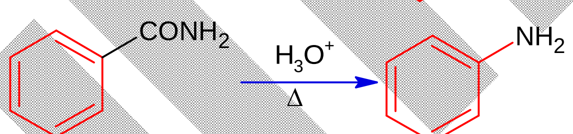
$$\lambda = 8\pi a_0$$

Q87. The number of the correct reaction(s) among the following is__________.

(A) 

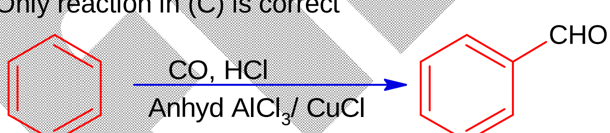
(B) 

(C) 

(D) 

Ans. 1

Sol. Only reaction in (C) is correct



Gatterman-Koch reaction

Q88. 2.5g of a non-volatile, non-electrolyte is dissolved in 100g of water at 25°C. The solution showed a boiling point elevation by 2°C. Assuming the solute concentration is negligible with respect to the solvent concentration, the vapour pressure of the resulting aqueous solution is______mm of Hg(nearest integer)
[Given: Molal boiling point elevation constant of water (K_b) =0.52 Kg mol^{-1}1atm pressure = 760 mm of Hg, molar mass of water = 18 g mol^{-1}]

Ans. 707

Sol.

$$\Delta T = K_b m$$

$$2 = 0.52 \times m$$

$$m = \frac{2}{52}$$

$$\frac{\Delta P}{P^0} = X_{solute} = \frac{n_{solute}}{n_{solvent}}$$
$$\Delta P = 760 \times \frac{2/0.52}{1000/18}$$
$$\Delta P = \frac{760 \times 2 \times 18}{0.52 \times 1000}$$
$$\Delta P = 52.615 \text{ mm of Hg}$$
$$P_{solution} = 760 - 52.615$$
$$= 707.385$$
$$= 707$$

Q89. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond- energies of C-H,C-C,C=C,H-H are 414 kJ, 347kJ, 615kJ and 435kJ respectively is ________kJ
Given 1506

Ans. 125

Sol.
$$CH_2 = CH_2 + H_2 \longrightarrow CH_3 - CH_3$$
$$\Delta H = BE(C = C) + BE(H - H) - BE(C - C) - BE\ 2(C - H)$$
$$615 + 435 - 347 - 2 \times 414$$
$$615 + 435 - 347 - 828$$
$$1050 - 1175$$
$$= -125\,kJ$$

Q90. Only 2 ml of $KMnO_4$ solution of unknown molarity is required to reach the end point of a titration of 20 mL of oxalic acid (2M) in acidic medium. The molarity of $KMnO_4$ solution should be ___________ M.

Ans. 8

Sol.
$$K\overset{+7}{Mn}O_4 \longrightarrow Mn^{+2} \qquad n = 5$$
$$C_2O_4^{-2} \longrightarrow 2CO_2 \qquad n = 2$$
$$n_1v_1M_1 = n_2v_2M_2$$
$$M_1 \times 2 \times 5 = 2 \times 20 \times 2$$
$$M_1 = \frac{80}{10}$$
$$M_1 = 8$$