# KVPY PAPER – 2012

# CLASS-XII

## PART-I

### ONE MARK QUESTIONS

### MATHEMATICS

**1.** Three children, each accompanied by a guardian, seek admission in a school. The principal wants to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guardian. In how many ways can this de done ?

(A) 60 (B) 90 (C) 120 (D) 180

**Sol.** **(B)**

No. of ways = ${}^6C_2 \times {}^4C_2 \times {}^2C_2 = 90$

**2.** In the real number system, the equation $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$ has :

(A) No solution (B) Exactly two distinct solutions

(C) Exactly four distinct solutions (D) Infinitely many solutions

**Sol.** **(D)**

$\sqrt{(x-1)+4-4\sqrt{x-1}} = 1-\sqrt{(x-1)+9-6\sqrt{x-1}}$

Put $\sqrt{x-1} = t$

$\sqrt{t^2+4-4t} = 1-\sqrt{t^2+9-6t}$

Squaring we get infinitely many solutions.

**3.** The maximum value M of $3^x + 5^x - 9^x + 15^x - 25^x$, as x varies over reals, satisfies :

(A) $3 < M < 5$ (B) $0 < M < 2$ (C) $9 < M < 25$ (D) $5 < M < 9$

**Sol.** **(B)**

Max. value = 1

**4.** Suppose two perpendicular tangents can be drawn from the origin to the circle $x^2 + y^2 - 6x - 2py + 17 = 0$, for some real p. Then |p| is equal to :

(A) 0 (B) 3 (C) 5 (D) 17

**Sol.** **(C)**

Director circle passes through origin so

Radius of director circle = $\sqrt{2}$ radius of circle

$$\sqrt{3^2+p^2} = \sqrt{2}\sqrt{3^2+P^2-17}$$

$$3^2 + p^2 = 2(3^2 + P^2 - 17)$$

$$0 = 3^2 + p^2 - 34$$

$$|P| = 5$$

**5.** Let a, b, c, d be numbers in the set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a-c)^2 + b - d$ is :

(A) 0 (B) 5 (C) 30 (D) 36

**Sol.** **(B)**

$ax + b = cx + d$

So far no solution $a = c$

so $(a-c)^2 + b - d = (a)^2 + 6 - 1 = 5$

**6.** Consider the conic

$ex^2 + \pi y^2 - 2e^2x - 2\pi^2 y + e^3 + \pi^3 = \pi e$. Suppose P is any point on the conic and $S_1$, $S_2$ are the foci of the conic, then the maximum value of $(PS_1 + PS_2)$ is :

(A) $\pi e$ (B) $\sqrt{\pi e}$ (C) $2\sqrt{\pi}$ (D) $2\sqrt{e}$

**Sol.** **(C)**

$ex^2 + \pi y^2 - 2e^2x - 2\pi^2 y + e^3 + \pi^3 = \pi e$

$$\frac{(x-e)^2}{\pi}+\frac{(y-\pi)^2}{e}=1$$

So $PS_1 + PS_2$ = Length of major axis = $2\sqrt{\pi}$

**7.** Let $f(x) = \dfrac{\sin(x-a)+\sin(x+a)}{\cos(x-a)-\cos(x+a)}$, then :

(A) $f(x + 2\pi) = f(x)$ but $f(x + \alpha) \neq f(x)$ for any $0 < \alpha < 2\pi$
(B) f is a strictly increasing function
(C) f is strictly decreasing function
(D) f is constant function

**Sol.** **(D)**

$$f(x) = \frac{\sin(x-a)+\sin(x+a)}{\cos(x-a)-\cos(x+a)}$$

$$= \frac{2\sin x.\cos a}{2\sin x.\sin a} = \tan a$$

So constant function

**8.** The value of $\tan 81° - \tan 63° - \tan 27° + \tan 9°$ is :

(A) 0 (B) 2 (C) 3 (D) 4

**Sol.** **(D)**

$\tan 81° - \tan 63° - \tan 27° - \tan$

$$\frac{\sin 81°}{\cos 81°}+\frac{\sin 9°}{\cos 9°}-\left[\frac{\sin 63°}{\cos 63°}+\frac{\sin 27°}{\cos 27°}\right]$$

$$\frac{\sin 90°}{\cos 81°.\cos 9°}-\left[\frac{\sin 90°}{\cos 63°.\cos 27°}\right]$$

$$\frac{2}{\sin 18°}-\frac{2}{\sin 54°}$$

$$2\left[\frac{1}{\sin 18°}-\frac{1}{\sin 59°}\right]$$

$$2\left[\frac{\sin 54°-\sin 18°}{\sin 18°\sin 54°}\right]=2\left[\frac{2\cos 36°.\sin 18°}{\sin 18°.\sin 54°}\right]=4$$

**9.** The mid-point of the domain of the function $f(x)=\sqrt{4-\sqrt{2x+5}}$ for real x is :

(A) $\frac{1}{4}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{5}$

**Sol.** **(B)**

$$f(x) = \sqrt{4-\sqrt{2x+5}}$$

$$x \geq -\frac{5}{2}$$

$$2x + 5 < 16$$

$$x \leq \frac{11}{2}$$

$$-\frac{5}{2} \leq x \leq \frac{11}{2}$$

$\Rightarrow$ Mid-point of domain = $\frac{3}{2}$

**10.** Let n be a natural number and let a be a real number. The number of zeros of $x^{2n+1}-(2n+1)x+a=0$ in the interval [–1, 1] is :

(A) 2 if $a > 0$ (B) 2 if $a < 0$
(C) At most one for every value of a (D) At least three for every value of a

**Sol.** **(C)**

Polynomial function is monotonic.

**11.** Let f : R → R be the function
$f(x) = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - a_3)(x - a_1)$
with $a_1, a_2, a_3 \in \mathbb{R}$. Then $f(x) \geq 0$ if and only if :
(A) at least two of $a_1, a_2, a_3$ are equal (B) $a_1 = a_2 = a_3$
(C) $a_1, a_2, a_3$ are all distinct (D) $a_1, a_2, a_3$ are all positive and distinct

**Sol.** **(B)**
$3x^2 - 2(a_1 + a_2 + a_3)x + a_1a_2 + a_2a_3 + a_3a_1$
$4[a_1 + a_2 + a_3]^2 - 4[3(a_1a_2 + a_2a_3 + a_3a_4)] \leq 0$
$(a_1 + a_2 + a_3)^2 - 3(a_1a_2) \leq 0$
$a_1^2 + a_2^2 + a_3^2 - a_1a_2 - a_2a_3 - a_3a_1 \leq 0$
$2[(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2] \leq 0$

**12.** The value $\dfrac{\int_0^{\pi/2}(\sin x)^{\sqrt{2}+1}dx}{\int_0^{\pi/2}(\sin x)^{\sqrt{2}-1}dx}$ is :

(A) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ (B) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (C) $\dfrac{\sqrt{2}+1}{\sqrt{2}}$ (D) $2-\sqrt{2}$

**Sol.** **(D)**

**13.** The value $\int_{-2012}^{2012}(\sin(x^3) + x^5 + 1)dx$ is :

(A) 2012 (B) 2013 (C) 0 (D) 4024

**Sol.** **(D)**

$$\int_{-2012}^{2012}[\sin(x^3) + x^5 + 1]dx$$

$$= \int_{-2012}^{2012}\left(\sin(x^3) + x^5\right) + \int_{-2012}^{2012}(1)dx$$

$= 0 + 4024$

**14.** Let [x] and {x} be the integer part and fractional part of a real number x respectively. The value of the integral $\int_0^5[x]\{x\}dx$ is :

(A) 5/2 (B) 5 (C) 34.5 (D) 35.5

**Sol.** **(B)**

$$\int_0^5[x]\{x\}dx = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = 5$$

**15.** Let $S_n = \sum_{k=1}^{n}k$ denote the sum of the first n positive integers. The numbers $S_1$, $s_2$, $S_3$,....., $S_{99}$ are written on 99 cards. The probability of drawing a card with an even number written on it is :

(A) $\dfrac{1}{2}$ (B) $\dfrac{49}{100}$ (C) $\dfrac{49}{99}$ (D) $\dfrac{48}{99}$

**Sol.** **(C)**
$S_1 \rightarrow$ odd
$S_2 \rightarrow$ odd
$S_3 \rightarrow$ even
$S_4 \rightarrow$ eve
$S_5 \rightarrow$ odd
$S_6 \rightarrow$ odd
$S_7 \rightarrow$ even
So 49 cards will be even
So probability = $\dfrac{49}{99}$

**16.** A purse contains 4 copper coins and 3 silver coins. A second purse contain 6 copper coins and 4 silver coins. A purse is chosen randomly and a coin is taken out of it. What is the probability that it is a copper coin ?

(A) $\frac{41}{70}$ (B) $\frac{31}{70}$ (C) $\frac{27}{70}$ (D) $\frac{1}{3}$

**Sol.** **(A)**

$$\text{Required probability} = \frac{1}{2}\left[\frac{4}{7}+\frac{6}{10}\right]$$
$$= \frac{1}{2}\left[\frac{4}{7}+\frac{3}{5}\right]$$
$$= \frac{1}{2}\left[\frac{41}{35}\right] = \frac{41}{70}$$

**17.** Let H be the orthocenter of an acute-angled triangle ABC and O be its circumcentre. Then

$\vec{HA}+\vec{HB}+\vec{HC}$

(A) Is equal to $\vec{HO}$ (B) Is equal to $3\vec{HO}$

(C) Is equal to $2\vec{HO}$ (D) Is not a scalar multiple of $\vec{HO}$ in general

**Sol.** **(C)**

**18.** The number of ordered pairs (m, n), where m, n $\in$ {1, 2, 3, ....., 50}, such that $6^m + 9^n$ is a multiple of 5 is :

(A) 1250 (B) 2500 (C) 625 (D) 500

**Sol.** **(B)**

We can choose m from (1, 2, ....., 50)
but we can choose n from (1, 3, 5, 7, ..... 49)
So that $6^m + 9^n$ is multiple of 5
So total number of pairs = 50 × 25 = 1250

**19.** Suppose $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indexed numbers is 3018, what is the sum of all numbers.

(A) 0 (B) 1509 (C) 3018 (D) 6036

**Sol.** **(D)**

$a_1, a_2, a_3, \ldots\ldots, a_{2012}$ must be in an A.P.
and $a_{2012} + a_2 = a_1$
So difference of these terms = 0
All numbers are equal.
So sum of all number = 2 × 3018 = 6036.

**20.** Let S = {1, 2, 3,.....,n} and
A = {(a, b) |1 ≤ a, b ≤ n} = S × S. A subset B of A is said to be a good subset if (x, x) $\in$ B for every x $\in$ S. Then the number of good subsets of A is :

(A) 1 (B) $2^n$ (C) $2^{n(n-1)}$ (D) $2^{n^2}$

**Sol.** **(C)**

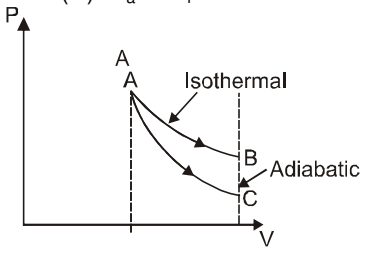
## PHYSICS

**21.** An ideal monatomic gas expands to twice its volume. If the process is isothermal, the magnitude of work done by the gas is $W_i$. If the process is adiabatic, the magnitude of work done by the gas is $W_a$. Which of the following is true ?

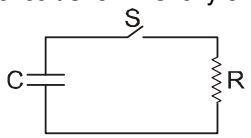
(A) $W_i = W_a > 0$ (B) $W_i > W_a = 0$ (C) $W_i > W_a > 0$ (D) $W_a > W_i = 0$

**Sol.** **(B)**

Area under the curve P vs V gives work done by gas and expansion work done by gas is positive.
$W_i > W_a > 0$



**22.** The capacitor of capacitance C is the circuit shown is fully charged initially. Resistance is R.




After the switch S is closed, the time taken to reduce the stored energy in the capacitor to half its initial value is :

(A) $RC_2$ (B) RC (C) ln2 2RC ln 2 (D) $\frac{RC\ln 2}{2}$

**Sol.** **(D)**

Let initial charge on capacitor be $Q_0$.

$$Q = Q_0\, e^{-t/CR}$$

Given , $$U = \frac{U_i}{2}$$

$$\Rightarrow \quad \frac{Q^2}{2c} = \frac{1}{2}\frac{Q_0{}^2}{X2c}$$

$$\Rightarrow \quad Q = \frac{Q_0}{\sqrt{2}}$$

$$\Rightarrow \quad e^{-t/CR} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad t = \frac{CR\ln 2}{2}$$

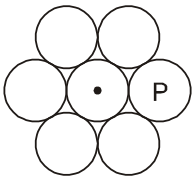
**23.** A liquid drop placed on a horizontal plane has a near spherical shape (slightly flattened due to gravity). Let R be the radius of its largest horizontal section. A small disturbance cause the drop to vibrate with frequency v about its equilibrium shape. By dimensional analysis the ratio. $\frac{v}{\sqrt{\sigma/\rho R^3}}$ can be (Here σ is density, g is acceleration due to gravity, and k is an arbitrary dimensionless constant.)

(A) $k\rho gR^2/\sigma$ (B) $k\rho R^3/g\sigma$ (C) $k\rho R^2/g\sigma$ (D) $k\rho/g\sigma$

**Sol.** **(A)**

$\frac{U}{\sqrt{\frac{6}{\delta R^3}}}$ is a dimensionless quality and the $\frac{K\delta gR^2}{6}$ is also a dimensionally quantity.

**24.** Seven identical coins are rigidly arranged on a flat table in the pattern shown below so that each coin touches its neighbors. Each coin is a thin disc of mass m and radius r. Note that the moment of inertia of an individual coin about an axis passing through centre and perpendicular to the plane of the coin is $mr^2/2$.




The moment of inertia of the system of seven coins about an axis that passes through the point P(the centre of the coin positioned directly to the right of the central coin) and perpendicular to the plane of the coins is :

(A) $\frac{55}{2}mr^2$ (B) $\frac{127}{2}mr^2$

(C) $\frac{111}{2}mr^2$ (D) $55\ mr^2$

**Sol.** **(C)**

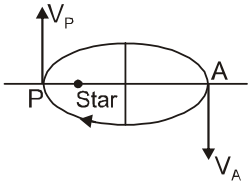
$$I_{Cm} = \left(\frac{mr^2}{2} + 4mr^2\right)\times 6 + \frac{mr^2}{2}$$

$$= \frac{55mr^2}{2}$$

using parallel axis theorem

$$I_P = \frac{55mr^2}{2} + 28mr^2 = \frac{111mr^2}{2}$$

**25.** A planet orbits in an elliptical path of eccentricity e round a massive star considered fixed at one of the foci. The point in space where it is closest to the star is denoted by P and the point where it is farthest is denoted by A. Let $v_p$ and $v_A$ be the respective speeds at P and A.Then




(A) $\frac{V_P}{V_A}=\frac{1+e}{1-e}$ (B) $\frac{V_P}{V_A}=1$ (C) $\frac{V_P}{V_A}=\frac{1+e^2}{1-e}$

**Sol.** **(A)**

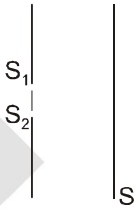
From conservation of Angular momentum about centre of star.

$$mV_P(a-ae) = mV_A(a+ae)$$

$$\Rightarrow \quad \frac{V_P}{V_A} = \frac{1+e}{1-e}$$

**26.** In a young's double slit experiment the intensity of light at each slit is $I_0$. Interference pattern is observed along a direction parallel to the line $S_1$ $S_2$ on screen S.

The minimum, maximum, and the intensity averaged over the entire screen are respectively.




(A) 0, $4I_0$, $2I_0$ (B) $I_0$, $2I_0$, $3I_0/2$

(C) 0, $4I_0I_0$ (D) 0, $2I_0$,$I_0$

**Sol.** **(A)**

$$I_r = I_0 + I_0 + 2\sqrt{I_0^2}\cos\phi$$

$$= 4I_0\cos^2\frac{\phi}{2}$$

$$I_{min} = 0$$

$$I_{max} = 4I_0$$

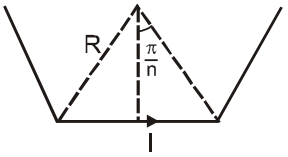
$$\text{Average Intensity} = 2I_0$$

**27.** A loop carrying current i has the shape of a regular polygon of n sides. If R is the distance from the centre to any vertex, then the magnitude of the magnetic induction vector $\vec{B}$ at the centre of the loop is :

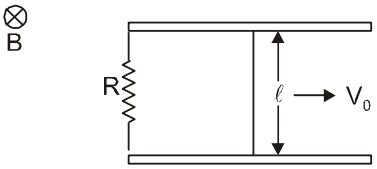
(A) $n\frac{\mu_0 I}{2\pi R}\tan\frac{\pi}{n}$ (B) $n\frac{\mu_0 I}{2\pi R}\tan\frac{2\pi}{n}$ (C) $\frac{\mu_0 I}{2R}$ (D) $\frac{\mu_0 I}{\pi R}\tan\frac{\pi}{n}$

**Sol.** **(A)**

$$B = \frac{n\mu_0 I\, 2\sin(\pi/n)}{4\pi R\cos(\pi/n)} = \frac{n\mu_0 I\tan(\pi/n)}{2\pi R}$$




**28.** A conduction rod of mass m and length $\ell$ is free to move without friction on two parallel long conducting rails, as shown below. There is a resistance R across the rails. In the entire space around, there is a uniform magnetic field B normal to the plane of the rod and rails. The rod is given an impulsive velocity $V_0$.




Finally, the initial energy $\frac{1}{2}mv_0^2$

(A) Will be converted fully into heat energy in the resistor

(B) Will enable rod to continue to move with velocity $v_0$ since the rails are frictionless

(C) Will be converted fully into magnetic energy due to induced current

(D) Will be converted into the work done against the magnetic field

**Sol.** **(A)**

Initial kinetic energy will be fully converted into heat energy in the resistor.

**29.** A steady current I flows through a wire of radius r, length L and resistivity ρ. The current produces heat in the wire. The rate of heat loss in a wire is proportional to its surface area. The steady temperature of the wire is independent of :

(A) L (B) r (C) I (D) ρ

**Sol.** **(A)**

$i^2R$ = Rate of heat loss through radiation

$$\Rightarrow \quad i^2 \frac{\rho l}{\pi r^2} \propto \pi r^2 l.\,(\theta - \theta_0)$$

$$\Rightarrow \quad \theta - \theta \propto \frac{i^2\rho}{\pi^2 r^4}$$

**30.** The ratio of the speed of sound to the average speed of an air molecule at 300k and I atmospheric pressure is close to :

(A) 1 (B) $\sqrt{300}$ (C) $\sqrt{1/300}$ (D) 300

**Sol.** **(A)**

$$V_S = \sqrt{\frac{\gamma RT}{m_0}}$$

$$<V> = \sqrt{\frac{8}{\pi}\frac{RT}{m_0}}$$

$$\therefore \quad \frac{V_S}{<V>} = \sqrt{\frac{\gamma.\pi}{8}} = \sqrt{0.55}$$

Nearest option is A

**31.** In one model of the electron, the electron of mass $m_e$ is through to be a uniformly charged shell of radius R and total charge e, whose electrostatic energy E is equivalent to its mass $m_e$ via Einsetein's mass energy relation $E = m_eC^2$. In this model, R is approximately ($m_e = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m.s$^{-1}$, ¼ $\pi\varepsilon_0 = 9 \times 10^9$ Farads.m$^{-1}$, magnitude of the electron charge = $1.6 \times 10^{-19}$ C)

(A) $1.4 \times 10^{-15}$ m (B) $2 \times 10^{-13}$ m (C) $5.3 \times 10^{-11}$ m (D) $2.8 \times 10^{-35}$ m

**Sol.** **(A)**

$$\frac{Ke^2}{2R} = m_ec^2$$

$$\Rightarrow \quad R = \frac{Ke^2}{2m_ec^2} = 1.407 \times 10^{-15}\text{m}$$

**32.** A body is executing simple harmonic motion of amplitude a and period T about the equilibrium position x = 0. Large numbers of snapshots are taken at random of this body in motion. The probability of the body beiong found in a very small interval x to x + |dx| is highest at :

(A) $x = \pm a$ (B) $x = 0$ (C) $x = \pm a/2$ (D) $x = \pm a/\sqrt{2}$

**Sol.** **(A)**

Probability of finding body will be max. where speed is minimum.

**33.** Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is held at a temperature of 100ºC while the other one is kept at 0ºC. If the two are brought into contact, then, assuming no heat loss to the environment, the final temperature that they will reach is :

(A) 50ºC (B) more than 50ºC (C) less than 50ºC (D) 0ºC

**Sol.** **(B)**

$$\text{Temp} \propto \frac{1}{\text{Heat capacity}}$$

With no change in heat capacity eq. temp. would be $\theta = 50°C$.

As heat capacity increase with temperature $\theta > 50°$

**34.** A particle is acted upon by a force given by $F = -\alpha x^3 - \beta x^4$ where $\alpha$ and $\beta$ are positive constants. At the point x = 0, the particle is :

(A) in stable equilibrium (B) in unstable equilibrium

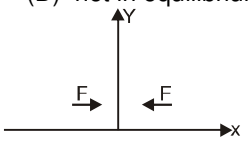
(C) in neutral equilibrium (D) not in equilibrium

**Sol.** **(A)**

For $x > 0 : F < 0$

$x < 0 : F > 0$

$\therefore$ stable equation



**35.** The potential energy of a point particle is given by the expression $V(x) = \alpha x + \beta \sin(x / \gamma)$. A dimensionless combination of the constants $\alpha$, $\beta$ and $\gamma$ is :

(A) $\alpha/\beta\gamma$ (B) $\alpha^2/\beta\gamma$ (C) $\gamma/\alpha\beta$ (D) $\alpha\gamma/\beta$

**Sol.** **(D)**

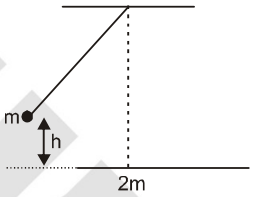
$$[\alpha] = \frac{[V]}{[x]}$$

$$[\beta] = [V]$$

$$[\gamma] = [x]$$

$$\left[\frac{\alpha\gamma}{\beta}\right] = \frac{[V]}{[x]} \cdot \frac{1}{[V]} \cdot [x] = 1$$

**36.** A ball of mass m suspended from a rigid support by an inextensible massless string is released from a height h above its lowest point. At its lowest point it collides elastically with a block of mass 2m at rest on a frictionless surface. Neglect the dimensions of the ball and the block. After the collision the ball rises to a maximum height of :




(A) h/3 (B) h/2

(C) h/8 (D) h/9

**Sol.** **(D)**

Velocity of ball after collision

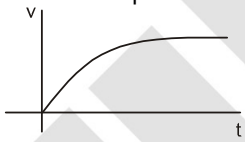
$$V_1 = \frac{m - 2m}{m + 2m}\sqrt{2gh}$$

$$= -\frac{\sqrt{2gh}}{3}$$

$\therefore$ Maximum height reached is given by

$$h_{max} = \frac{v_1^2}{2g} = \frac{h}{9}$$

**37.** A particle released from rest is falling through a thick fluid under gravity. The fluid exerts a resistive force on the particle proportional to the square of its speed. Which one of the following graphs best depicts the variation of its speed v with time t ?

(A) 

(B) 

(C) 

(D) 

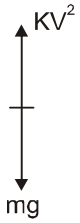

**Sol.** **(A)**

Force acting on particle.

$$a = \frac{dv}{dt} = mg - kv^2$$

Hence speed of particle will increase and becomes constant

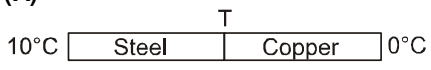
when $kv^2 = mg \Rightarrow v = \sqrt{\frac{mg}{k}}$




**38.** A cylindrical steel rod of length 0.10 m and thermal conductivity 50 $W.m^{-1}.K^{-1}$ is welded end to end to copper rod of thermal conductivity 400 $W.m^{-1}.K^{-1}$ and of the same area of cross section but 0.20 m long. The free end of the steel rod is maintained at 100ºC and that of the copper rod at 0ºC. Assuming that the rods are perfectly insulated from the surrounding, the temperature at the junction of the two rods is :

(A) 20º (B) 30º (C) 40º (D) 50º

**Sol.** **(A)**




$$\frac{T - 100}{R_S} = \frac{T - 0}{R_C}$$

$$\Rightarrow \quad \frac{T - 100}{T} = -\frac{R_S}{R_C} = -4$$

$$\therefore \quad T = 20°C$$

**39.** A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. The half lives of X and Y are 40000 years and 20 years respectively. In a certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is $4 \times 10^{20}$, the number of Y nuclei present in it is :

(A) $2 \times 10^{17}$ (B) $2 \times 10^{20}$ (C) $4 \times 10^{23}$ (D) $4 \times 10^{20}$

**Sol.** **(A)**

$$X \xrightarrow{\lambda_x} Y \xrightarrow{\lambda_y} Z$$

$$\lambda_x N_x = \lambda_y N_y$$

$$\Rightarrow \quad N_y = \frac{\lambda_x}{\lambda_y} . N_x$$

$$= \frac{(t_{1/2})_y}{(t_{1/2})_x} \cdot N_x$$

$$= \frac{20}{40000} \times 4 \times 10^{20}$$

$$= 2 \times 10^{17}$$

**40.** An unpolarized beam of light of intensity $I_0$ passes through two linear plarizers making an angle of 30º with respect to each other. The emergent beam will have an intensity :

(A) $\frac{3I_0}{4}$ (B) $\frac{\sqrt{3}I_0}{4}$ (C) $\frac{3I_0}{8}$ (D) $\frac{I_0}{8}$

**Sol.** **(A)**

Malus's law

$$I = I_0 \cos^2 \theta$$

$$I = \frac{3I_0}{4}$$

## CHEMISTRY

**41.** Among the following, the species with the highest bond order is :

(A) $O_2$ (B) $F_2$ (C) $O_2^+$ (D) $F_2^-$

**Sol.** **(C)**

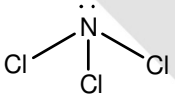
$$\sigma 1s^2 \, \sigma 1s^{*2} \, \sigma 1s^2 \, \sigma 2s^{*2} \, \sigma 2p_z^2 \left[ \begin{matrix} \pi 2p_x^2 \\ \pi 2p_y^2 \end{matrix} \right] \left[ \begin{matrix} \pi 2p_x^{*0} \\ \pi 2p_y^{*1} \end{matrix} \right]$$

$$BO = \frac{n - n^*}{2} = \frac{10 - 5}{2} = \frac{5}{2} = 2.5$$

**42.** The molecule with non-zero dipole moment is :

(A) $BCl_3$ (B) $BeCl_2$ (C) $CCl_4$ (D) $NCl_3$

**Sol.** **(D)**



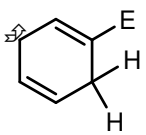

**43.** For a one-electron atom, the set of allowed quantum numbers is :

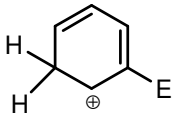
(A) $n = 1, \ell = 0, m_\ell = 0, m_s = +1/2$ (B) $n = 1, \ell = 1, m_\ell = 0, m_s = +1/2$

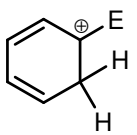
(C) $n = 1, \ell = 0, m_\ell = -1, m_s = -1/2$ (D) $n = 1, \ell = 1, m_\ell = 1, m_s = -1/2$

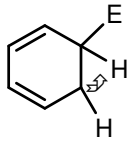
**Sol.** **(A)**

**44.** In the reaction of benzene with an electrophile E+, the structure of the intermediate σ-complex can be represented as :

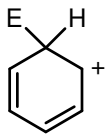
(A) 


(B) 

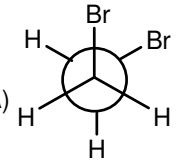

(C) 

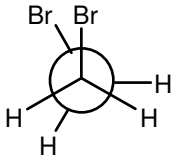

(D) 

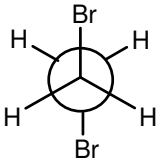
**Sol.** **(D)**

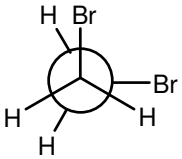


**45.** The most stable conformation of 2,3-dibromobutane is :

(A) 

(B) 

(C) 

(D) 

**Sol.** **(C)**

Staggered is most stable.

**46.** Typical electronic energy gaps in molecules are about 1.0 eV. In terms of temperature, the gap is closest to :

(A) $10^2$ K (B) $10^4$ K (C) $10^3$ K (D) $10^5$ K

**Sol.** **(C)**

$K_BT \approx 0.025$ eV at 0 K

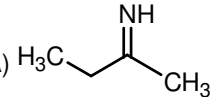
So, $\frac{K_BT}{K_BT_2} = \frac{0.025}{1.00}$

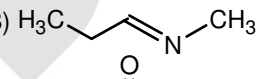
$T_2 = 4 \times T_1$

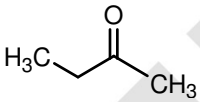
$= 4 \times 273 = 1092 \approx 10^3$ K

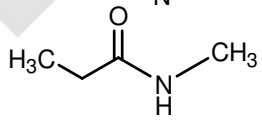
**47.** The major final product in the following reaction is :

$$CH_3CH_2CN \xrightarrow[(2)H_3O^+]{(1)CH_3MgBr}$$

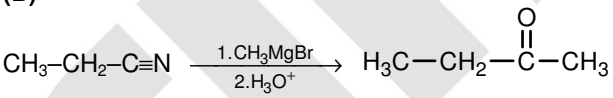
(A) 

(B) 

(C) 

(D) 

**Sol.** **(B)**



**48.** A zero-order reaction, A → Product, with an initial concentration $[A]_0$ has a half-life of 0.2 s. If one starts with the concentration $2[A]_0$, then the half-life is :

(A) 0.1 s (B) 0.4 s (C) 0.2 s (D) 0.8 s

**Sol.** **(B)**

Half-life of zero order reaction is given by

$$t_{50} = \frac{C_0}{2k}, \quad t_{50} \propto C_0$$

Hence half life will get doubled.

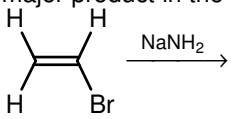
**49.** The isoelectronic pair of ions is :

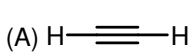
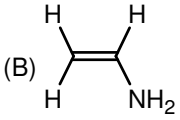
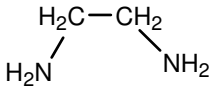
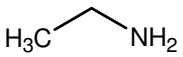
(A) $Sc^{2+}$ and $V^{3+}$ (B) $Mn^{3+}$ and $Fe^{2+}$ (C) $Mn^{2+}$ and $Fe^{3+}$ (D) Ni3+ and $Fe^{2+}$

**Sol.** **(C)**

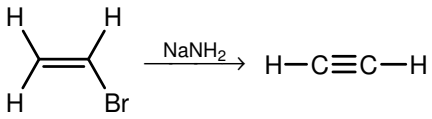
Both have 24 electrons.

**50.** The major product in the following reaction is :

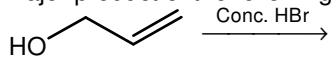


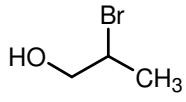
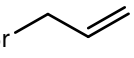
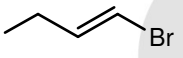
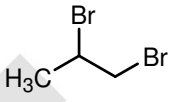
(A)  (B)  (C)  (D) 

**Sol.** **(A)**

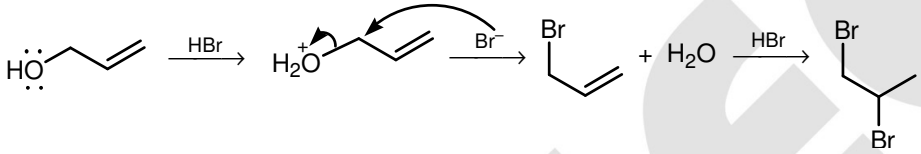


**51.** The major product of the following reaction is :

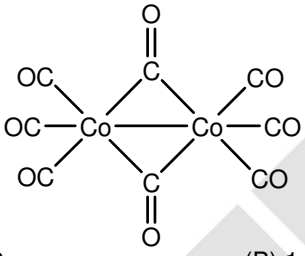


(A)  (B)  (C)  (D) 

**Sol.** **(D)**



**52.** The oxidation state of cobalt in the following molecule is :



(A) 3 (B) 1 (C) 2 (D) 0

**Sol.** **(D)**

All ligands are neutral.

**53.** The pKa of a weak acid is 5.85. The concentrations of the acid and its conjugate base are equal at a pH of:

(A) 6.85 (B) 5.85 (C) 4.85 (D) 7.85

**Sol.** **(B)**

pH = pKa = 5.85

$$Ka = \frac{[H^+][A^-]}{[AA]}$$

$Ka = [H^+]$

pKa = pH

**54.** For a tetrahedral complex $[MCl_4]^{2-}$, the spin-only magnetic moments is 3.83 BM. The element M is :

(A) Co (B) Cu (C) Mn (D) Fe

**Sol.** **(A)**

$$3.83 \text{ BM} = \sqrt{n(n+2)}\,\text{BM}$$

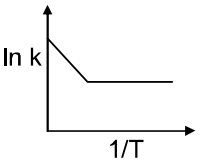
$$3.83 = \sqrt{n(n+2)}$$

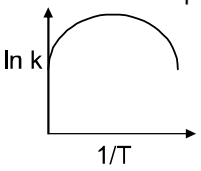
$$n = 3$$

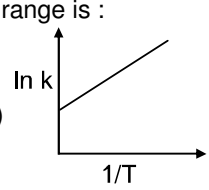
So $M^{2+}$ has three unpaired electrons.

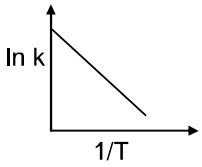
Hence metal is Co.

**55.** Among the following graphs showing variation of rate (k) with temperature (T) for a reaction, the one that exhibits Arrhenius behavior over the entire temperature range is :

(A) 


(B) 


(C) 


(D) 

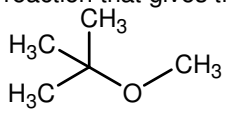

**Sol.** **(D)**

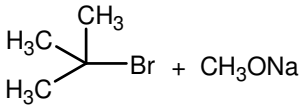
$K = Ae^{-E/RT}$

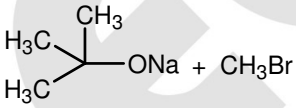
$\ln K = \ln A - \frac{E}{RT}$

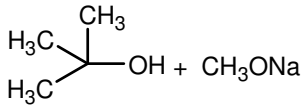
So the ln K vs $\frac{1}{T}$ is linear with –ve slope & +ve intercept.

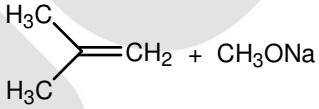
**56.** The reaction that gives the following molecule as the major product is :



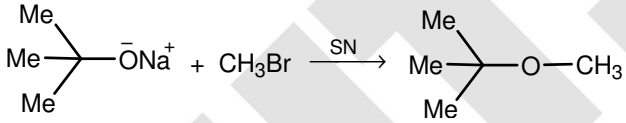
(A) 

(B) 

(C) 

(D) 

**Sol.** **(B)**



**57.** The C–O bond length in CO, $CO_2$, and $CO_3^{2-}$ follows the order :

(A) $CO < CO_2 < CO_3^{2-}$ (B) $CO_2 < CO_3^{2-} < CO$ (C) $CO > CO_2 > CO_3^{2-}$ (D) $CO_3^{2-} < CO_2 < CO$

**Sol.** **(A)**

As decided by bond order

$CO < CO_2 < CO_3^{2-}$

**58.** The equilibrium constant for the following reactions are $K_1$ and $K_2$, respectively.

$2P_{(g)} + 3Cl_{2(g)} \rightleftharpoons 2PCl_{3(g)}$

$PCl_{3(g)} + Cl_{2(g)} \rightleftharpoons PCl_{5(g)}$

Then the equilibrium constant for the reaction

$2P_{(g)} + 5Cl_{2(g)} \rightleftharpoons 2PCl_{5(g)}$ is

(A) $K_1K_2$ (B) $K_1K_2^2$ (C) $K_1^2K_2^2$ (D) $K_1^2K_2$

**Sol.** **(B)**

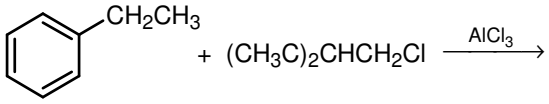
$2P_{(g)} + 3Cl_{2(g)} \rightleftharpoons 2PCl_{3(g)}$; $K_1$

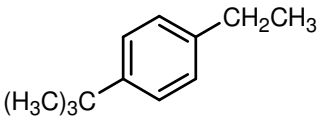
$PCl_{3(g)} + Cl_{2(g)} \rightleftharpoons PCl_{5(g)}$; $K_2$

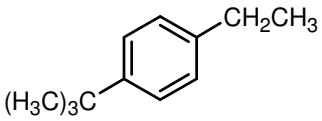
Multiplying the 2nd equation by 2 and adding it with 1st

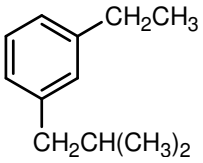
We get $2P_{(g)} + 5Cl_{2(g)} \rightleftharpoons 2PCl_{5(g)}$ $K = K_1 \times K_2^2$

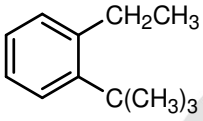
**59.** The major product of the following reaction is :



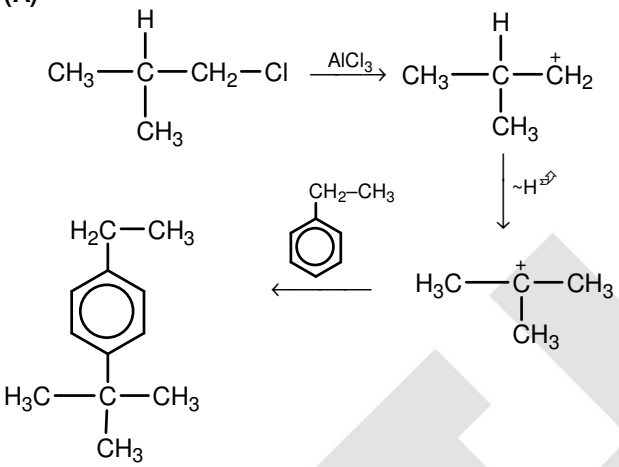
(A) 

(B) 

(C) 

(D) 

**Sol.** **(A)**



**60.** Doping silicon with boron produces a :
(A) n-type semiconductor (B) metallic conductor (C) p-type semiconductor (D) insulator

**Sol.** **(C)**
Doping with trivalent impuring produces p-type semiconductor.

## BIOLOGY

**61.** The disorders that arise when the immune system destroys 'self' cells are called autoimmune disorders. Which of the following would be classified under this ?
(A) rheumatioid arthritis (B) asthama (C) rhinitis (D) eczema

**Sol.** **(A)**
Rhematioid arthritis is an autoimmune disease while rhinits, asthma and eczema are inflammatory disease of, mucous membrane inside nose, bronchioles and epidermis respectively.

**62.** Which of the following class of immunoglobulin can trigger the complement cascade ?
(A) IgA (B) IgM (C) IgD (D) IgE

**Sol.** **(B)**
Classical complement pathway comprising of 21 different serum proteins primarily activated by antibody molecules of either IgG or IgM.

**63.** Diabetes insipidus is due to :
(A) hypersecretion of vasopressin (B) hyposecretion of insulin
(C) hypersecretion of insulin (D) hyposecretion of vasopressin

**Sol.** **(D)**
Vasopressin or ADH hormone induce reabsorption of water in DCT and collecting duct of nephron. It's hyposecretion may cause frequent urination which is also called Diabetes insipidus.

**64.** Fossils are most often in which kind of rocks ?
(A) meteorites (B) sedimentary rocks (C) igneous rocks (D) metamorphic rocks

**Sol.** **(B)**
Fossils are most often found in Sedimentary kind of rocks.

**65.** Peptic ulcers are caused by :
(A) a fungus, Candida albicans (B) a virus, cytomegalovirus
(C) a parasite, Trypanosoma brucei (D) a bacterium Helicobacter pylori
**Sol.** **(D)**
Helicobacter pylori previously named compylobacter pyloridis, is a gram – negative bacteria found in stomach, causes peptic ulcers.

**66.** Transfer RNA (tRNA)
(A) is present in the ribosomes and provides structural integrity
(B) usually has clover leaf-like structure
(C) carries genetic information from DNA to ribosomes
(D) codes for proteins
**Sol.** **(B)**
rRNA is present in the ribosomes and mRNA carries genetic information from DNA and codes for protein, while tRNA carries amino acids at site of protein synthesis and also have clover leaf-like sturcture

**67.** Some animals excrete uric acid in urine (uricotelic) as it requires very little water loss. Which animals among the following are most likely to be uricotelic ?
(A) fishes (B) amphibians (C) birds (D) mammals
**Sol.** **(C)**
Birds are uricotelic.

**68.** A ripe mango, kept with unripe mangoes causes their ripening. This is due to the release of a gaseous plant hormone.
(A) auxin (B) gibberlin (C) cytokinine (D) ethylene
**Sol.** **(D)**
Ethylene is only gaseous plant hormone that induce ripening of fruit.

**69.** Human chromosomes undergo structural changes during the cell cycle. Chromosomal structure can be best visualized if a chromosome is isolated from a cell at :
(A) GI phase (B) S phase (C) G2 phase (D) M phase
**Sol.** **(D)**
Chromosomal structure can be best visualized in metaphase of M Phase .

**70.** By which of the following mechanisms is glucose reabsorbed from the glomerular filtrate by the kideny tubule ?
(A) osmosis (B) diffusion (C) active transport (D) passive transport
**Sol.** **(B)**
In kidney tubule PCT, 100% glucose from glomerular filtrate is reabsorbed by diffusion.

**71.** In mammals, the hormones secreted by the pituitary, the master gland, is itself regulated by the pituitary, the master gland, is itself regulated by :
(A) hypothalamus (B) median cortex (C) pineal gland (D) cerebrum
**Sol.** **(A)**
Secretion of hormones by the pituitary gland is regulated by a part of brain “hypothalamus”.

**72.** Which of the following is true TCA cycle in eukaryotes ?
(A) takes place in mitochondrion (B) produces no ATP
(C) takes place in Golgi complex (D) independent of electron transport chain
**Sol.** **(A)**
Kreb Cycle/TCA cycle in eukaryotes takes place in mitochondria.

**73.** A hormone molecule binds to a specific protein on the plasma membrane inducing a signal. The protein it binds to is called :
(A) ligand (B) antibody (C) receptor (D) histone
**Sol.** **(C)**
Proteins on plasma membrane to which hormone molecules bind is called receptor.

**74.** DNA mutations that do not cause any functional change in the protein product are known as :
(A) nonsense mutations (B) missense mutations (C) deletion mutations (D) silent mutations

**Sol.** **(D)**
Silent mutations are DNA mutations that do not result in a change to the amino acid sequence of a protein.

**75.** Plant roots are usually devoid of chlorophyll and cannot perform photosynthesis. However, there are exceptions. Which of the following plant root can perform photosynthesis ?
(A) Arabidopsis (B) Tinospora (C) Rice (D) Hibiscus

**Sol.** **(B)**
Some plants develop green, chlorophyll containing roots along with colourless roots, these green roots perform photosynthesis
e.g. – Trapa and Tinospora

**76.** Vitamin A deficiency leads to night-blindness. Which of the following is the reason for the disease ?
(A) rod cells are not converted to cone cells
(B) rhodopsin pigment of rod cells is defective
(C) melanin pigment is not synthesized in cone cells
(D) cornea of eye gets dried

**Sol.** **(B)**
Vitamins A is structural component of visual pigment. "Rhodopsin", vitamin A deficiency may leads to night blindness because rhodopsin pigment of rod cells become defective.

**77.** In Dengue virus infection, patients often develop haemorrhagic fever due to internal bleeding. This happens due to the reduction of :
(A) platelets (B) RBCs (C) WBCs (D) lymphocytes

**Sol.** **(A)**
Haemorrhagic fever in dengue virus infection may cause internal bleeding due to the reduction of platelets.

**78.** If the sequence of bases in sense strand of DNA is 5' – GTTCATCG-3', then the sequence of bases in its RNA transcript would be :
(A) 5'-GTTCATCG-3' (B) 5'-GUUCAUCG-3'
(C) 5'-CAAGTAGC-3' (D) 5'-CAAGUAGC-3'

**Sol.** **(B)**
Transcribed RNA is exactly like that of the sense strand of DNA
DNA $\rightarrow 5^1$ - GTTCATCG – 3'
RNA $\rightarrow 5^1$ – GUUCAUCG – 3'

**79.** A reflex action is a quick involuntary response to stimulus. Which of the following is an example of BOTH, unconditioned and conditioned reflex ?
(A) knee Jerk reflex
(B) secretion of saliva in response to the aroma of food
(C) sneezing reflex
(D) contraction of pupil in response to bright light

**Sol.** **(B)**
Secretion of saliva in response to the aroma of food could be both unconditional and conditional reflex.

**80.** In a food chain such as grass $\rightarrow$ deer $\rightarrow$ lion, the energy cost of respiration as a proportion of total assimilated energy at each level would be :
(A) 60% - 30% - 20% (B) 20% - 30% - 60% (C) 20%- 60%- 30% (D) 30%- 30% - 30%

**Sol.** **(D)**
According to 10% low at each tropic leve 90% of assimilated energy is lost in respiration and only 10% is pass on the next level so respiration energy loss would be constant at each tropic level.

# PART-II
## TWO MARKS QUESTIONS

## MATHEMATICS

**81.** Suppose a, b, c are real numbers, and each of the equations $x^2 + 2ax + b^2 = 0$ and $x^2 + 2bx + c^2 = 0$ has two distinct real roots. Then the equation $x^2 + 2cx + a^2 = 0$ has :
(A) two distinct positive real roots (B) two equal roots
(C) one positive and one negative root (D) no real roots

**Sol.** **(D)**

$x^2 + 2ax + b^2 = 0 \Rightarrow D_1 > 0 \Rightarrow 4a^2 - 4b^2 > 0 \Rightarrow a^2 - b^2 > 0 \Rightarrow a^2 > b^2$

$x^2 + 2bx + c^2 = 0 \Rightarrow D_2 > 0 \Rightarrow 4b^2 - 4c^2 > 0 \Rightarrow b^2 > c^2 \Rightarrow a^2 > b^2 > c^2$

& $x^2 + 2cx + a^2 = 0 \Rightarrow D_3 = 4(c^2 - a^2) < 0$

**82.** The coefficient of $x^{2012}$ in $\frac{1+x}{(1+x^2)(1-x)}$ is :

(A) 2010 (B) 2011 (C) 2012 (D) 2013

**Sol.** **(Option are wrong) Answer is 1**

$$\frac{(1+x)^2}{(1+x^2)(1-x^2)} = \frac{(1+x)^2}{1-x^4}$$

Coefficient of $x^{2012}$ is 1.

**83.** Let (x, y) be a variable point on the curve $4x^2 + 9y^2 - 8x - 36y + 15 = 0$. Then min $(x^2 - 2x + y^2 - 4y + 5)$ + max $(x^2 - 2x + y^2 - 4y + 5)$ is :

(A) $\frac{325}{36}$ (B) $\frac{36}{325}$ (C) $\frac{13}{25}$ (D) $\frac{25}{13}$

**Sol.** **(A)**

$4x^2 + 9y^2 - 8x - 36y + 15 = 0$

$4(x^2 - 2x + 1) - 4 + 9(y^2 - 4y + 4) - 36 + 15 = 0$

$4(x-1)^2 + 9(y-2)^2 = 40 - 15 = 25$

$$\frac{(x-1)^2}{(5/2)^2} + \frac{(y-2)^2}{(5/3)^2} = 1$$

$$x - 1 = \frac{5}{2}\cos\theta$$

$$y - 2 = \frac{5}{3}\sin\theta$$

$$\text{Min.}\left((x-1)^2 + (y-2)^2\right) + \text{Max.}\left((x-1)^2 + (y-2)^2\right)$$

$$\text{Min.}\left(25\frac{\cos^2\theta}{4} + \frac{25\sin^2\theta}{9}\right) + \text{Max.}\left(25\frac{\cos^2\theta}{4} + 25\frac{\sin^2\theta}{9}\right)$$

$$= \frac{25}{9} + \frac{25}{4} = \frac{325}{36}$$

**84.** The sum of all $x \in [0, \pi]$ which satisfy the equation $\sin x + \frac{1}{2}\cos x = \sin^2\left(x + \frac{\pi}{4}\right)$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\pi$ (D) $2\pi$

**Sol.** **(C)**

$$\sin x + \frac{1}{2}\cos x = \sin^2(x + \pi/4)$$

$$2\sin x + \cos x = 2\sin^2(x + \pi/4)$$

$$2\sin x + \cos x = 1 + \sin 2x$$

$2\sin x + \cos x - 1 - 2\sin x\cos x = 0$

$2\sin x(1 - \cos x) - 1(1 - \cos x) = 0$

$(2\sin x - 1)(1 - \cos x) = 0$

$\sin x = \frac{1}{2}$, $\cos x = 1$

$x = 30°, 150°, x = 0$

**85.** A polynomial P(x) with real coefficients has the property that P"(x) ≠ 0 for all x. Suppose P(0) = 1 and P'(0) = –1. What can you say about P(1) ?

(A) $P(1) \geq 0$ (B) $P(1) \leq 0$ (C) $P(1) \neq 0$ (D) $-1/2 < P(1) < 1/2$

**Sol.** **(Question is wrong)**

**86** Define a sequence $\langle a_n \rangle$ by $a_1 = 5$, $a_n = a_1 a_2 \ldots\ldots a_{n-1} + 4$ for $n > 1$. Then $\lim_{n\to\infty} \frac{\sqrt{a_n}}{a_{n-1}}$

(A) equals $\frac{1}{2}$ (B) equal 1 (C) equals $\frac{2}{5}$ (D) does not exist

**Sol.** **(C)**

$$\lim_{n\to\infty} \frac{\sqrt{a_n}}{a_{n-1}} = \lim_{n\to\infty} \sqrt{\frac{a_1 a_2 \ldots a_{n-1} + 4}{(a_{n-1})^2}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{a_1 a_2 \ldots\ldots a_{n-2}}{a_{n-1}} + \frac{4}{(a_{n-1})^2}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{1}{1 + \frac{4}{a_1 a_2 \ldots a_{n-2}}} + \frac{4}{(a_{n-1})^2}} = 1$$

**87.** The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, where $a > 0$, is :

(A) $\pi$ (B) $a\pi$ (C) $\frac{\pi}{2}$ (D) $2\pi$

**Sol.** **(C)**

$$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx = \int_0^{\pi} \cos^2 x \, dx = \frac{\pi}{2}$$

**88.** Consider

$L = \sqrt[3]{2012} + \sqrt[3]{2013} + \ldots\ldots + \sqrt[3]{3011}$

$R = \sqrt[3]{2013} + \sqrt[3]{2014} + \ldots\ldots + \sqrt[3]{3012}$

and $I = \int_{2012}^{3012} \sqrt[3]{x} dx$.

Then

(A) $L + R < 2I$ (B) $L + R = 2I$ (C) $L + R > 2I$ (D) $\sqrt{LR} = I$

**Sol.** **(A)**

$$\frac{L+R}{2} < I \Rightarrow L + R < 2I$$

**89.** A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let P(K) be the probability of scoring at least K points. The larges value of K such that P(K) > ½ is :

(A) 14 (B) 15 (C) 16 (D) 17

**Sol.** **(B)**

**90.** Let $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$. Let.

$f^1(x) = f(x), f^2(x) = f(f(x))$ and generally

$f^n(x) = f(f^{n-1}(x))$ for $n > 1$.

Let $P = f^1(2) f^2(3) f^3(4) f^4(5)$

Which of the following is a multiple of P

(A) 125 (B) 375 (C) 250 (D) 147

**Sol.** **(B)**

$f'(2) = 3$, $f^2(3) = -3$, $f^3(4) = -5/3$, $f^4(5) = 5$

$P = 75$

**91.** The total energy of a black body radiation source in collected for five minutes and used to heat water. The temperature of the water increases form 10.0°C to 11.0°C. The absolute temperature of the black body is doubled and its surface area halved and the experiment repeated for the same time. Which of the following statements would be most nearly correct ?
(A) The temperature of the water would increase from 10.0°C to a final temperature of 12°C
(B) The temperature of the water would increase from 10.0°C to a final temperature of 18°C
(C) The temperature of the water would increase from 10.0°C to a final temperature of 14°C
(D) The temperature of the water would increase from 10.0°C to a final temperature of 11°C

**Sol.** **(B)**

$$H = ms\Delta T$$

$$\Rightarrow \quad \sigma A T_0^4 = ms \times 1 \qquad \text{.....(i)}$$

$$\sigma \frac{A}{2}(2T_0)^2 = ms\,\Delta T \qquad \text{.....(ii)}$$

from (i) & (ii)

$$\Delta T = 8°C$$

**92.** A small asteroid is orbiting around the sun in a circular orbit of radius $r_0$ with speed $V_0$. A rocket is launched from the asteroid with speed $V = \alpha V_0$, where V is the speed relative to the sun. The highest value of a for which the rocket will remain bound to the solar system (ignoring gravity due to the asteroid and effects of other planets).
(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) 1

**Sol.** **(A)**

$$V_0 = \sqrt{\frac{GM}{r_0}} \qquad \text{.....(i)}$$

For rocket to remain bounded with solar system, mechanical energy < 0

$$\Rightarrow \quad \frac{1}{2}mv^2 - \frac{GMm}{r_0} \le 0 \qquad \text{.....(ii)}$$

$$\Rightarrow \quad \alpha \le \sqrt{2}$$

$$\alpha_{max} = \sqrt{2}$$

**93.** A radioactive nucleus A has a single decay mode with half life $\tau_A$. Another radioactive nucleus B has two decay modes 1 and 2. If decay mode 2 were absent, the half life of B would have been $\tau_A/2$. If decay mode 1 were absent, the half life of B would have been 2 $\tau_A$. If the actual half life of B is $\tau_B$. then the ratio $\tau_B/\tau_A$ is :
(A) 3/7 (B) 7/2 (C) 7/3 (D) 1

**Sol.** **(A)**

$$\tau_B = \frac{\ln 2}{\lambda_{eff.}}$$

$$\lambda_{eff.} = \lambda_1 + \lambda_2$$

$$= \frac{2\ln 2}{\tau_A} + \frac{\ln 2}{3\tau_A}$$

$$\Rightarrow \quad \tau_B = \frac{\ln 2}{\frac{2\ln 2}{\tau_A} + \frac{\ln 2}{3\tau_A}}$$

$$= \tau_A\left(\frac{3}{7}\right) \Rightarrow \frac{\tau_B}{\tau_A} = \frac{3}{7}$$

**94.** A stream of photons having energy 3 eV each impinges on a potassium surface. The work function of potassium is 2.3 eV. The emerging photo-electrons are slowed down by a copper plate placed 5 mm away. If the potential difference between the two metal plates is 1 V, the maximum distance the electrons can move away from the potassium surface before being turned back is :
(A) 3.5 mm (B) 1.5 mm (C) 2.5 mm (D) 5.0 mm

**Sol.** **(A)**

$$K_{max} = 3eV - 2.3\ eV$$

$= 0.7$ eV

using work energy theorem

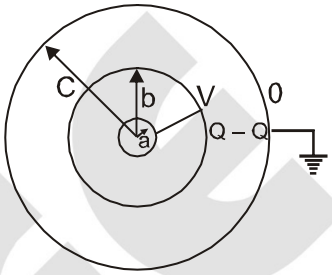
$$0.7 = 200\,\ell$$

$$\ell = \frac{0.7}{200} = 3.5 \text{ mm.}$$

**95.** Consider three concentric metallic spheres A, B and C of radii a, b, c respectively where a < b < c. A and B are connected whereas C is grounded. The potential of the middle sphere B is raised to V then the charge on the sphere C is :

(A) $-4\pi\varepsilon_0 V\frac{bc}{c-b}$ (B) $+4\pi\varepsilon_0 V\frac{bc}{c-b}$ (C) $-4\pi\varepsilon_0 V\frac{ac}{c-a}$ (D) Zero

**Sol.** **(A)**




$$\Delta V_{BC} = Q\left(\frac{1}{4\pi\varepsilon_0 b} - \frac{1}{4\pi\varepsilon_0 c}\right)$$

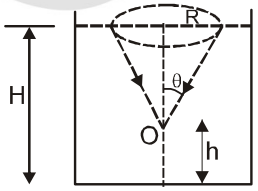
$$Q = \frac{4\pi\varepsilon_0 Vbc}{c-b}$$

Change on sphere c = – Q = $\frac{-4\pi\varepsilon_0 Vbc}{c-b}$

**96.** On a bright sunny day a diver of height h stands at the bottom of a lake of depth H. Looking upwards, he can see objects outside the lake in a circular region of radius R. Beyond this circle he sees the images of objects lying on the floor of the lake. If refractive index of water is 4/3, then the value of R is :

(A) $3(H-h)/\sqrt{7}$ (B) $3h\sqrt{7}$ (C) $(H-h)/\sqrt{7/3}$ (D) $(H-h)/\sqrt{5/3}$

**Sol.** **(A)**

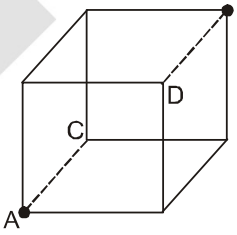



$$\sin\theta = \frac{1}{\mu} = \frac{1}{4}\times 3 = \frac{3}{4}$$

$$\tan\theta = \frac{3}{\sqrt{7}} = \frac{R}{H-h}$$

$$\Rightarrow \quad R = \frac{(H-h)3}{\sqrt{7}}$$

**97.** As shown in the figure below, a cube is formed with ten identical resistances R (thick lines) and two shorting wires (dotted lines) along the arms AC and BD.



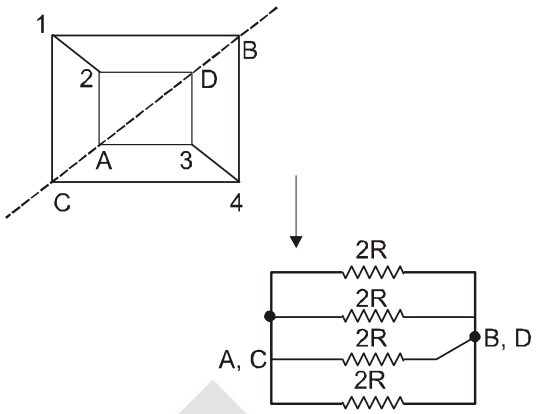

Resistance between point A and B is :

(A) R/2 (B) 5R/6 (C) 3R/4 (D) R

**Sol.** **(A)**

Transforming the circuit

$\therefore$ Effective resistance $= \frac{R}{2}$



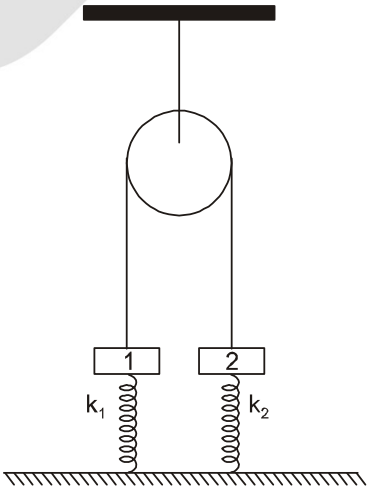

**98.** A standing wave in a pipe with a length L = 1.2 m is described by

$$y(x,t) = y_0 \sin[(2\pi/L)x]\sin[(2\pi/L)x + \pi/4]$$

Based on above information, which one of the following statement is **incorrect.**
(Speed of sound in air is 300 m.s.$^{-1}$)
(A) The pipe is closed at both ends
(B) The wavelength of the wave could be 1.2 m
(C) There could be a node at x = 0 and antinode at x = L/2
(D) The frequency of the fundamental mode of vibrations is 137.5 Hz

**Sol.** **(**Wrong DATA**)**

**99.** Two blocks (1 and 2) of equals mass m are connected by an ideal string (see figure below) over a frictionless pulley. The bocks are attached to the ground by springs having spring constants $k_1$ and $k_2$ such that $k_1 > k_2$.
Initial, both springs are unscratched. The block 1 is slowly pulled down a distance x and released. Just after the release the possible values of magnitudes of the accelerations of the blocks $a_1$ and $a_2$ can be :




(A) either $\left(a_1 = a_2 = \frac{(k_1+k_2)x}{2m}\right)$ or $\left(a_1 = \frac{k_1x}{m} - g \text{ and } a_2 = \frac{k_2x}{m} + g\right)$

(B) $\left(a_1 = a_2 = \frac{(k_1+k_2)x}{2m}\right)$ only

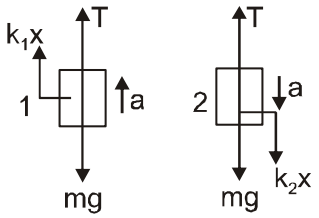
(C) $\left(a_1 = a_2 = \frac{(k_1-k_2)x}{2m}\right)$ only

(D) either $\left(a_1 = a_2 = \frac{(k_1-k_2)x}{2m}\right)$ or $\left(a_1 = a_2 = \frac{(k_1k_2)x}{(k_1+k_2)m} - g\right)$

**Sol.** **(A)**

(i) If $K_1x > mg$

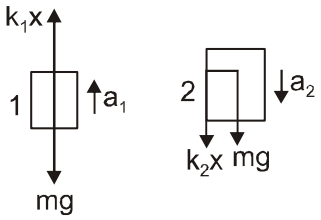
$\therefore$ $a = \frac{(k_1+k_2)x}{2m}$



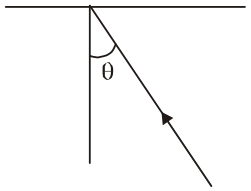

(ii) If $k_1x > mg$

$a_1 = \frac{k_1x}{m} - g$

$a_2 = \frac{k_2x}{m} + g$



**100.** A simple pendulum is released from rest at the horizontally stretched position. When the string makes an angle $\theta$ with the vertical, the angle $\phi$ which the acceleration vector of the bob makes with the string is given by :




(A) $\phi = 0$ (B) $\varphi = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$ (C) $\varphi = \tan^{-1}(2\tan\theta)$ (D) $\phi = \pi/2$

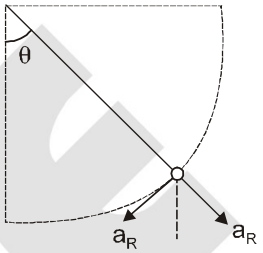
**Sol.** **(B)**

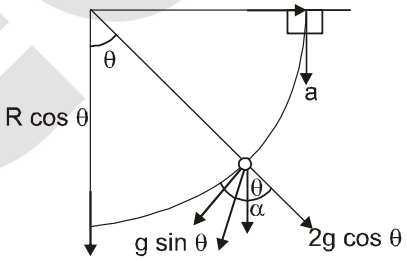
$$a_T = g\sin\theta$$

$$a_R = \frac{v^2}{R} = \frac{2gR\cos\theta}{R} = 2g\cos\theta$$

$$\therefore \quad \tan\varphi = \frac{g\sin\theta}{2g\cos\theta}$$

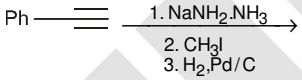
$$= \frac{\tan\theta}{2}$$

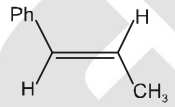


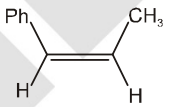



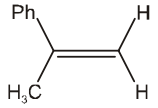

## CHEMISTRY

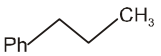
**101.** The final major product obtained in the following sequence of reaction is :



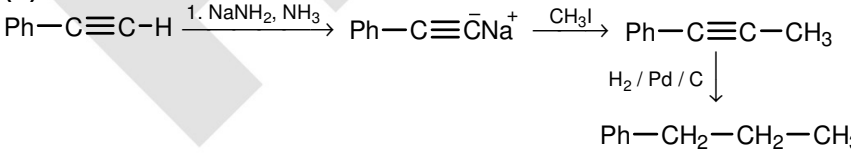

(A) 


(B) 


(C) 


(D) 


**Sol.** **(D)**




**102.** In the DNA of E. Coli the mole ratio of adenine to cytosine is 0.7. If the number of moles of adenine in the DNA is 350000, the number of moles of guanine is equal to :

(A) 350000 (B) 500000 (C) 225000 (D) 700000

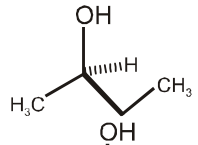
**Sol.** **(B)**

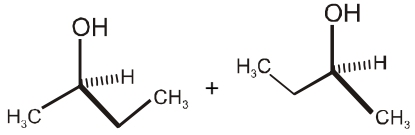
$$\frac{A}{C} = 0.7$$

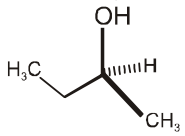
$$C = \frac{A}{0.7} = \frac{350000}{0.7} = 50{,}000$$

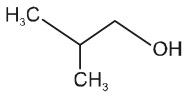
$$A = G$$

**103.** (R)-2-bromobutane upon treatment with aq. NaOH gives :

(A) 

(B) 

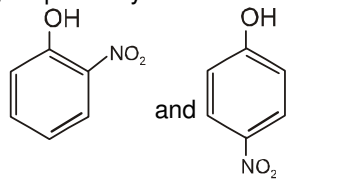
(C) 

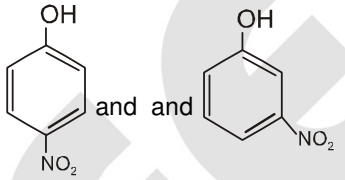
(D) 

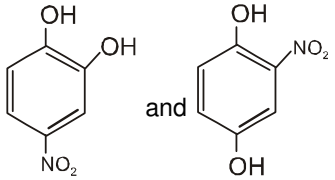
**Sol.** **(C)**

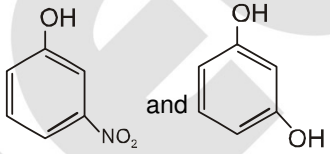
(R)-2-bromobutane upon treatment with aq. NaOH will give (S)-2-bromobutane with $S_{N}2$ (inversion).

**104.** Phenol on treatment with dil. $HNO_3$ gives two products P and Q. P is steam volatile but Q is not. P and Q are, respectively :

(A) 

(B) 

(C) 

(D) 

**Sol.** **(A)**

P is steam volatile due to intramolecular H-bonding.

**105.** A metal is irradiated with light of wavelength 660 nm. Given that the work function of the metal is 1.0 eV, the de Broglie wavelength of the ejected electron is close to :

(A) $6.6 \times 10^{-7}$ m (B) $8.9 \times 10^{-11}$ m (C) $1.3 \times 10^{-9}$ m (D) $6.6 \times 10^{-13}$ m

**Sol.** **(C)**

$$\frac{hc}{\lambda} = \frac{12400}{600} = 1.88eV$$

$$h\upsilon = \phi + KE$$

$$KE = (1.88 - 1)\ eV$$

$$= 0.88\ eV$$

$$l = \sqrt{\frac{150e}{E(eV)}}$$

$$= \sqrt{\frac{150}{0.88}} = \sqrt{170.45}\ Å$$

$$= 13\ Å = 1.3 \times 10^{-9}\ m$$

**106.** The inter-planar spacing between the (2 2 1) planes ofa cubic lattice of length 450 pm is :

(A) 50 pm (B) 150 pm (C) 300 pm (D) 450 pm

**Sol.** **(B)**

$$d = \frac{a}{\sqrt{h^2 + k^2 + \ell^2}} = \frac{450}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{450}{\sqrt{9}} = 150pm$$

**107.** The $\Delta H$ for vaporization of a liquid is 20 kJ/mol. Assuming ideal behaviour, the change in internal energy for the vaporization of 1 mol of the liquid at 60ºC and 1 bar is close to :

(A) 13.2 kJ/mol (B) 17.2 kJ/mol (C) 19.5 kJ/mol (D) 20.0 kJ/mol

**Sol.** **(B)**

$$H = E + PV$$

$$\Delta H = \Delta E + \Delta(PV)$$

$$\Delta H = \Delta E + P\Delta V \text{ (as P = constant)}$$

$$\Delta E = \Delta H - P\Delta V$$

$= \Delta H - n_g RT$

$= \left(20 - \frac{1\times 8.314\times 333}{1000}\right)$ kJ/mol

$= 20 - 2.7687$ kJ/mol

$= 17.2$ kJ/mol

**108.** Among the following, the species that is both tetrahedral and diamagnetic is :

(A) $[NiCl_4]^{2-}$ (B) $[Ni(CN)_4]^{2-}$ (C) $Ni(CO)_4$ (D) $[Ni(H_2O)_6]^{2+}$

**Sol. (C)**

$Ni(CO)_4$ oxidation state of Ni is zero so hybridization is $sp^3$ without any unpaired electron.

**109.** Three moles of an ideal gas expands reversibly under isothermal condition from 2 L to 20 L at 300 K. The amount of heat-change (in kJ/mol) in the process is :

(A) 0 (B) 7.2 (C) 10.2 (D) 17.2

**Sol. (D)**

$\Delta E = Q + w$

$\Delta E = 0$ under given condition

$Q = -w$

$= -\left(-nRT\ln\frac{V_2}{V_1}\right)$

$= 3 \times 8.314 \times 300 \times 2.303 \log \frac{20}{2}$

$= 17.232$ kJ/mol

**110.** The following data are obtained for a reaction, $X + Y \rightarrow$ Products.

| Expt. | $[X_0]$/mol | $[Y_0]$/mol | rate/mol $L^{-1}s^{-1}$ |
|---|---|---|---|
| 1. | 0.25 | 0.25 | $1.0 \times 10^{-6}$ |
| 2. | 0.50 | 0.25 | $4.0 \times 10^{-6}$ |
| 3. | 0.25 | 0.50 | $8.0 \times 10^{-6}$ |

The overall order of the reaction is :

(A) 2 (B) 4 (C) 3 (D) 5

**Sol. (C)**

As rate of reaction gets 8 times by doubling the conc. of y while keeping conc. of x constant so order of reaction with respect to y is 3. Similarly, keeping Y constant rate gets quadrupled on doubling conc. of X.
Hence order of reaction with respect to X is 2.
So over all order of reaction is 5.

## BIOLOGY

**111.** When hydrogen peroxide is applied on the wound as a disinfectant, there is frothing at the site of injury, which is due to the presence of an enzyme in the skin that sues hydrogen peroxide as a substrate to produce :

(A) hydrogen (B) carbon Dioxide (C) water (D) oxygen

**Sol. (D)**

Peroxidase enzyme use hydrogen peroxide as a substrate to produce oxygen that cause frothing at the site of injury.

**112.** Persons suffering from hypertension (high blood pressure) are advised a low-salt diet because :

(A) more slat is absorbed in the body of a patient with hypertension
(B) high salt leads to water retention in the blood that further increases the blood pressure
(C) high salt increases nerve conduction and increases
(D) high salt causes adrenaline release that increases blood pressure

**Sol. (B)**

High salt leads to water retention in the blood that increases blood volume and further increases the blood pressure.

**113.** Insectivorous plants that mostly grow on swampy soil use insects as a source of :

(A) carbon (B) nitrogen (C) phosphorous (D) magnesium

**Sol. (B)**

Insectivorous plants use insects as source of nitrogen.

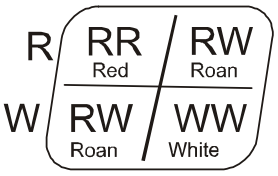
**114.** In cattle, the coat colour red and white are two dominant traits, which express equally in F1 to produce roan (red and white colour in equal proportion). If F1 progeny are self-bred, the resulting progeny in F2 will have phenotypic ratio (red : roan : white) is :

(A) 1 : 1 : 1 (B) 3 : 9 : 3 (C) 1 : 2 : 1 (D) 3 : 9 : 4

**Sol.** **(C)**

• Phenotypic Ratio (Red : Roan : White) is 1 : 2 : 1

• As red and white colour trait are codominant on selfing of $F_2$ generation – RW




**115.** The restriction endonuclease EcoR-I recongnises and cleaves DNA sequence as shown below :

5'-G A A T T C-3'

3' – C T T A A G-5'

What is the probable number of cleavage sites that can occur in a 10 kb long random DNA sequence ?

(A) 10 (B) 2 (C) 100 (D) 50

**Sol.** **()**

**116.** Which one of the following is true about enzyme catalysis ?

(A) the enzyme changes at the end of the reaction

(B) the activation barrier of the process is lower in the presence of an enzyme

(C) the rate of the reaction is retarded in the presence of an enzyme

(D) the rate of the reaction is independent of substrate concentration

**Sol.** **(B)**

Enzyme serve as biocatalyst, it increases rate of a biological reaction by decreasing activation energy.

**117.** Vibrio cholerae cuases cholera in humans. Ganga water was once used successfully to combat the infection. The possible reason could be :

(A) high salt content of Ganga water (B) low salt content of Ganga water

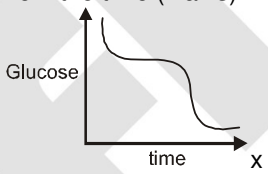
(C) presence of bacteriophages in Ganga water (D) presence of antibiotics in Ganga water

**Sol.** **(D)**

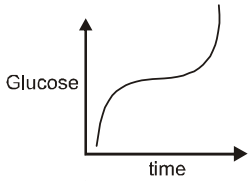
Ganga water may possibly contain antibiotic that may kill vibrio cholerae bacteria certainly.

**118.** When a person begins to fast, after some time glycogen stored in the liver is mobilized as a source of glucose. Which of the following graphs best represents the change of glucose level (y axis) in his blood, starting from the time (x axis) when he begins to fast ?

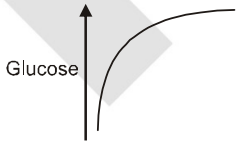
(A)



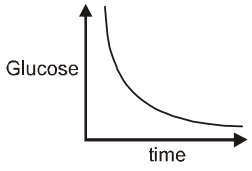

(B)




(C)




(D)




(A) (B) (C) (D)

**Sol.** **(A)**

On starting fast glucose level decreased and than remain constant as glucose began to mobilize from stored glycogen in liver and then further decreased.

**119.** The following sequence contains the open reading frame of a polypeptide. How many amino acids will the polypeptide consist of ?

5'-AGCATATGATCGTTTCTCTGCTTTGAACT-3'

(A) 4 (B) 2 (C) 10 (D) 7

**Sol.** **(D)**

An open reading fram (ORS) contain both initiating and stope codone, given ORS may possibly encode 7 amino acids.

**120.** Insects constitute the largest animals group on earth. About 25-30% of the insect species are known to be herbivores. In spite of such huge herbivore pressure, globally, green plants have persisted. One possible reason for this persistence is :
(A) food presence of insects has tended to change with time
(B) herbivore insects have become inefficient feeders of green plants
(C) herbivore population has been kept in control by predators
(D) decline in reproduction of herbivores with time

**Sol.** **(C)**
Herbivore population has bee kept in control by predators.

* * * * *